An Investigation of Flood Frequency Estimation Methods for the Upper Mississippi Basin

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September 2000

Acknowledgments

Dr. David Goldman of the Hydrologic Engineering Center produced this report. He received a considerable amount of help in obtaining data, selecting methods and in the form of review comments from others involved in the Upper Mississippi Flood Frequency Study. The Interagency Advisory Group (Mr. Kenneth Bullard, Dr. William Kirby (USGS), Mr. Alan Johnson (FEMA), Dr. Lesley Julian (NWS), Mr. Gregory Lowe (TVA) and Mr. Don Woodward (NRCS)) and the Technical Advisory Group (Dr. Jonathan Hosking (IBM Watson Research Center). Dr. William Lane (Consultant, Golden, Colorado), Dr. Kenneth Potter (University of Wisconsin), Dr. Jery Stedinger (Cornell University), Mr. Wilbert O. Thomas, Jr. (Michael Baker Corporation)) provided excellent method recommendations and review comments. The Corps of Engineers' districts (St. Paul, Rock Island, Omaha, Kansas City and St. Louis) and divisions (Mississippi Valley and Northwest) involved in the study provided data and excellent review comments. Special thanks to S. K. Nanda (Rock Island District) for instituting this investigation as part of the Upper Mississippi Flood Frequency study. Additional thanks to Dr. Jonathan Hosking (IBM, Watson Research Center) for providing the L-Moment estimation software, and to Dr. Timothy Cohn (US Geological Survey, Reston) for the expected moments algorithm software)

Executive Summary

- A reevaluation of the flood frequency estimates for the Upper Mississippi Basin has been instituted because: 1) there is about 30-years of additional data available since the last study; 2) there is some concern that the current federal guidelines are not applicable to the large basin of interest; and, 3) of the occurrence of the great flood of 1993.

- The investigation has involved a selection process where experts in flood frequency analysis (a Federal Interagency Advisory Group (IAG), Technical Advisory Group (TAG), and Corps of Engineers Districts and Division Offices) have had an opportunity to comment on proposed methods and preliminary results.

- An initial investigation of preliminary estimates of the peak annual 1-day stream flows was performed reflecting comments by the IAG regarding estimation procedures that might be used in the study. Regionalizing statistics, particularly regional skew, was not viewed by the IAG as being worthwhile given the diverse nature of hydrologic response occurring over the study area. Consequently, the initial study focused on only at-site estimation techniques.

- The initial investigation applied goodness-of-fit measures of quantile estimates in split sample tests to attempt to select a best flood distribution/estimation pairing. These tests were not definitive, in that the best method identified was a function of the measure used and the type of split sampling performed. Consequently, a sensitivity analysis was performed comparing predictions obtained from the Bulletin 17B guidelines and the log-normal distribution paired with a variety of estimation techniques (i.e., standard and L-moments, top-half censoring). The sensitivity analysis resulted in a maximum average difference of 10% between flood predictions at the (1/100) chance exceedance probability floods.

- The TAG suggested an alternative goodness of fit measure focusing the split sample testing on exceedances rather than quantiles. This was suggested because exceedances were the focus of the original goodness of fit tests used to establish Bulletin 17B. The results of this application were similar to the quantile comparisons and not definitive.

- The TAG was not totally convinced of the usefulness of the goodness of fit testing and recommended additional sensitivity analyses be performed to investigate the potential importance of regional shape estimation. As applied here, regional shape estimation involves estimating distribution parameters from the at-site mean and L-moment estimated coefficient of variation, and, substituting a regional shape parameter for the at-site value. In the case of the log-Pearson III distribution, regional shape estimation was accomplished by substituting a regional skew value for the adopted skew.

- The TAG's goal in considering this approach was to determine the difference between distributions obtained by Bulletin 17B and a regional shape approach. If the difference is not significant, then their preferred approach is log-Pearson III with a weighted skew coefficient. The IAG's preferences are the same, except they would prefer using the station skew.

- The region used to obtain the shape parameters or skew was determined using statistical techniques developed by Hosking and Wallis (1997). This analysis identified twenty of twenty-three stations in the study area as being part of the same region. The discordant stations, located at Anoka, St. Paul and Mankato, in Minnesota, may be influenced more by snow melt floods than the other stations. Further investigation of the appropriate flood distribution estimation methods to use for these stations is warranted.

- The regional shape estimation techniques were compared in a sensitivity analysis to the estimates obtained from Bulletin 17B. This resulted in a maximum absolute average difference between the flood distribution/estimation methods compared of about 10%-15% for the 1% flood. The sensitivity of stage prediction to the method chosen resulted in differences of a foot or more for the (1/25) year flood at a number of stations.

- A generalized least squares investigation of skew variation was not helpful in defining a regional skew pattern. Effectively, this investigation could not improve over an average skew assumption either for the purposed of skew weighting or in obtaining regionally consistent skew statistics.

- The TAG and IAG review of the comparisons done do not provide compelling evidence to deviate from application of the log-Pearson III distribution estimated by application of the method of moments to log-flows (see Appendix E). Computed maximum prediction differences of 10-15% at the 1% flood are not large considering the difficulty involved in measuring flood peaks, obtaining unregulated flows and converting these flows to peak stages.

- Flood quantiles are not constrained by the distribution/estimation methods to consistently vary along the study area rivers. The IAG and TAG proposed a simple smoothing algorithm where river reaches are identified based on the impact of major confluences on statistics; the mean and standard deviation are interpolated linearly between gaged locations within these reaches; and a weighted skew value is assumed constant within the reach.

- Some evidence of non-randomness was found both in statistical analyses performed in this investigation and by Olsen and Stakhiv (1999). However, the recommendation is to use the standard techniques applied in flood frequency analysis despite evidence for non-randomness at the gages in the study area.

- Other recommendations by the IAG and TAG related to the computation of flood quantiles are summarized in Appendix E.

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1. Introduction

The Corps of Engineers is involved in a reevaluation of the flood-frequency estimates for the main stem rivers in the Upper Mississippi Basin. The overall plan of study (Corps of Engineers, 1998) will involve a significant effort to develop the data and models needed for this evaluation.

The motivation for the reevaluation of the flood-frequency relationship estimates has been discussed at length elsewhere (e.g., see Corps of Engineers, 1998). Briefly, this motivation resulted from: 1) the significant additional period of record available since the last study (approximately 30-years of additional record); 2) the occurrence of the great flood of 1993; and finally, 3) the potential limitations of methods in Bulletin 17B (IACWD, 1982), the federal guidelines for performing flood frequency analysis, for application to the large basins involved in the Upper Mississippi Basin Study.

This report describes the results of a study where the methods described in the Bulletin 17B guidelines are evaluated in comparison with other flood frequency estimation techniques for application to large watersheds. The study was deemed to be necessary because the original studies used to develop the 17B methods focused on drainage areas significantly smaller than those of interest in the Upper Mississippi Basin. This 17B limitation is apparent from the regional skew map published in the guidelines which is recommended for applications to drainage areas less than 3,000 square miles.

The study was performed with the help of a number of peer review groups (see Table 1). The Technical Advisory Group and Interagency Advisory Group provided guidance on the methods to be applied and testing criteria. The Corps of Engineers districts and divisions reviewed results and aided in making final decisions on the selection methodology. The federal/state task force was involved in the review process and provided a perspective on the regulatory requirements that any proposed methodology would need to address.

Table 1.1: Peer review group membership

Group	Membership
interagency advisory	Kenneth Bullard (Bureau of Reclamation), Alan Johnson (Federal Emergency Management Agency), Lesley Julian (National Weather Service), William Kirby (Geological Survey), Donald Woodward (Natural Resource Conservation Service),Ming Tseng (Corps of Engineers);Greg Lowe (Tennessee Valley Authority)
technical advisory	Jon Hosking (IBM), William Lane (consultant), Kenneth Potter (University of Wisconsin), Jery Stedinger (Cornell University), Wilbert Thomas, (Michael Baker Jr., Inc.)
Corps of Engineers	Districts: St. Paul, Rock Island, Omaha, Kansas City, and St. Louis; Divisions: Mississippi Valley Division, Northwest Division; Head Quarters, Washington D.C.
Federal/State task force	Corps of Engineers, Bureau of Reclamation, Federal Emergency Management Agency, National Weather Service, U.S. Geological Survey, Natural Resources Conservation Service, Tennessee Valley Authority and States of Minnesota, Wisconsin, Iowa, Illinois, Missouri, Kansas, Nebraska

The investigation of flood frequency estimation methods evolved as investigation results were presented to the peer review groups and additional analyses suggested. Initially, a comparison of estimates obtained using IAG recommended methods was made with empirical distributions estimated from the data base described in section 2. Sections 3 and 4 describe the methods used in and the results of the comparative study. A review of the results by an independent peer group of experts in flood frequency analysis, the technical advisory group (TAG), suggested additional sensitivity analyses to perform given the comparative study results. Sections 5 and 6 provide a description of the methods recommended by the TAG and the results of the sensitivity analysis.

The flood flow frequency estimation methods investigated do not ensure flood quantile estimates that consistently vary along the study area rivers. Section 7 describes investigations into algorithms appropriate for obtaining regularly varying flood quantiles.

Stationarity or homogeneity of the observed data is a key assumption in a flood frequency analysis. The estimated unregulated flood record for the Upper Mississippi may deviate from this assumption either due to the influence of land use change, channel change or the climatic variability. Standard statistical tests were applied to the period of record to determine if any of these influences might cause a deviation from the standard assumption as is described in section 8. Section 9 provides some concluding remarks.

2. Data base

The goal in establishing the data base was to maximize the number of main-stem large area drainage basins available for the frequency distribution study. Maximizing the number of gages provides more opportunity to compare different frequency distributions to observed frequencies in the distribution selection procedure.

A 3,000 square mile minimum drainage area size was established to focus both on the importance of large drainage areas on the frequency analysis problem and examine drainage areas that exceeded those used in establishing the 17B guidelines. This minimum drainage area requirement necessarily limits the number of gages available. First, the number of large area basins that can be gaged is limited by topography and economics. Second, the records available at these gages are not homogenous for the most part, being influenced by regulation, channel modification and land use change. Consequently, a major effort is being instituted by the Corps to estimate the unregulated flows by accounting for these influences as part of the overall Upper Mississippi Basin study.

Preliminary estimates of unregulated flows have been developed by the Corps at the locations shown in Figure 2.1 and described in Table 2.1. Approximate corrections for the effects of regulation, levee failures and land use change have been made to the observations. The estimates are more reliable for the Mississippi Basin upstream of the Missouri River confluence, and correspondingly, less reliable for the Missouri River and downstream of the confluence with the Mississippi River.

An additional source of homogenous gage records was identified from the U.S. Geological Survey national stream gage data base (see Thomas and Eash, 1995) as is shown in Table 2.2. As can be seen from the table, the drainage areas involved here are considerably smaller than those chosen for the Corps study. However, the drainage areas do exceed the minimum proposed, and provide additional valuable information for evaluating probability distributions proposed for flood-frequency analysis.

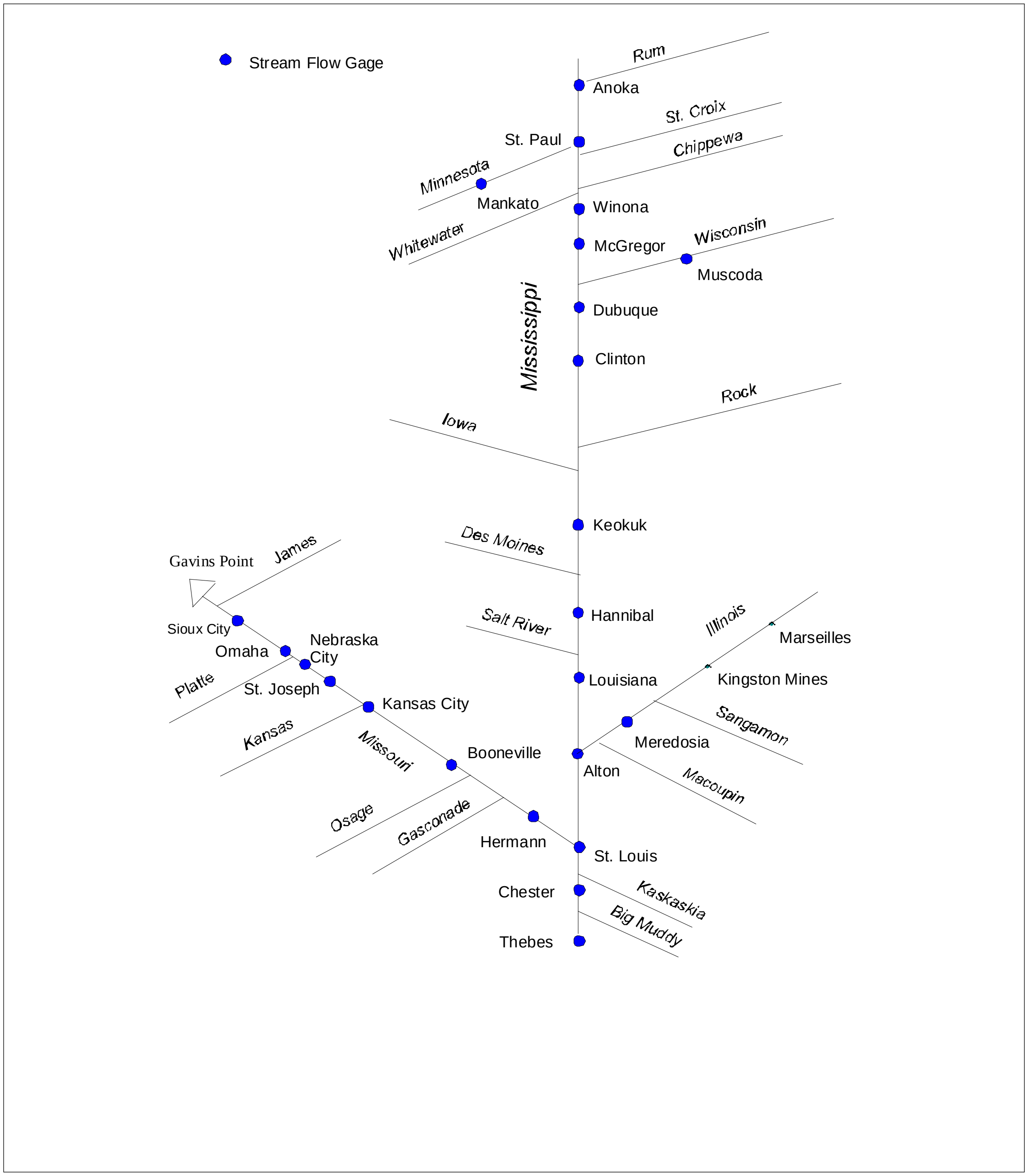

Figure 2.1: Schematic of main-stem gages where unregulated flows will be developed

Table 2.1: Main-stem gages

Location	River	*Drainage Area (sq mi)	Record length (years)	Period of Record
Annoka, Minnesota	Mississippi	19,600	65	1931-1995
Mankato, Minnesota	Minnesota	14,900	93	1903-1995
St. Paul, Minnesota	Mississippi	36,800	130	1867-1997
Winona, Minnesota	Mississippi	59,200	110	1885-1995
McGregor, Iowa	Mississippi	67,500	58	1937-1995
Muscoda, Iowa	Wisconsin	10,400	62	1915-1976
Sioux City, Iowa	Missouri	314,600 (35,120)	100	1898-1997
Omaha, Nebraska	Missouri	322,820 (43340)	100	1898-1997
Nebraska City, Nebraska	Missouri	414,420 (134940)	100	1898-1997
St. Joseph, Missouri	Missouri	420,300 (149860)	100	1898-1997
Kansas City, Missouri	Missouri	489,162 (209860)	100	1898-1997
Booneville, Missouri	Missouri	505,710 (226230)	100	1898-1997
Hermann, Missouri	Missouri	528,200 (248720)	100	1898-1997
Dubuque, Iowa	Mississippi	82,000	118	1879-1996
Clinton, Iowa	Mississippi	85,600	122	1875-1996
Keokuk, Iowa	Mississippi	119,000	118	1875-1996
Hannibal, Missouri	Mississippi	137,000	118	1879-1996
Meredosia, Illinois	Illinois	26,000	75	1921-1995
Louisiana, Missouri	Mississippi	140,700	118	1928-1995
Alton/Grafton, Illinois	Mississippi	171,300	68	**1928-1996
St. Louis, Missouri	Mississippi	697,013 (417520)	135	1861-1995
Chester, Illinois	Mississippi	708,563 (429120)	71	1926-1996
Thebes, Illinois	Mississippi	713,200 (433720)	64	1933-1996

*Drainage areas in ( ) are minus drainage area to Gavins Point
**Broken record

Table 2.2: Gage locations and record lengths for USGS data selected for study

Location	River	Drainage Area (sq mi)	Record length (years)	Period of Record
Conesville, Iowa	Cedar	7,785	55	1940-1994
Wapello, Iowa	Iowa	12,500	92	1903-1994
Augusta, Iowa	Skunk	4,303	80	1915-1994
Fort Dodge, Iowa	Des Moines	4,190	62	** 1914-1994
Stratford, Iowa	Des Moines	5,452	27	1968-1994
Van Meter, Iowa	Racoon	3,441	80	1915-1994
Scotland, South Dakota	James	3,898	66	1929-1994
Brookings, South Dakota	Big Sioux	20,653	41	1954-1994
Waterloo, Nebraska	Elkhorn	6,900	76	** 1899-1994
Beatrice, Nebraska	Big Blue	3,901	91	1902-1994
Sumner, Missouri	Grand	6,880	72	** 1922-1994

** Broken Record

3. Estimation methods and distributions for comparative study

3.1 Introduction

The interagency and technical advisory groups (IAG and TAG, Corps of Engineers, 1997) provided recommendations for the estimation techniques to be used in inferring the distributions of peak annual stream flows. The recommendations were made without any detailed knowledge of the data available, the quality of this data or the characteristics of the existing flood control system. Despite this, very important insights to the methods and approaches that needed to be used in inferring the appropriate frequency distributions can be gained from their recommendations. In particular, the estimation techniques recommended by the IAG and TAG to be investigated are: 1) the standard method of moments; 2) L-moments; 3) regression with censoring; and , 4) expected moments with censoring. These estimation techniques are to be applied with a set of suitable probability distributions.

Initially, regional analysis was not discussed as part of the estimation procedure. However, this issue was later addressed by the IAG. As is discussed in section 3.2, the IAG recommended against using regional analysis to obtain estimates of distribution parameters. The estimation methods described in section 3.3 only consider at-site (i.e., gage observations) estimation techniques for the comparative analysis. Section 3.4 describes the probability distributions that will be used with the various estimation techniques.

3.2 Regional analysis

The IAG recognized that annual peak floods are not due to a random pattern of storms and the basin response characteristics are not similar over the 713,200 square mile study area. The basin response characteristics of the watersheds differ greatly from the snowmelt driven floods occurring in the upper reaches of the study area to the combination of factors causing flood in St. Louis. Furthermore, the peak annual flood record exhibits significant inter-station correlation which reduces the effective independent period in the regional analysis (see Tables 3.1 and 3.2). Consequently, the IAG did not believe regional information would be of value in this study.

To appreciate the IAG concerns consider the ideal situation for the application of regional analysis. Assume the difference between the frequency statistics of the annual peak data observed for many watersheds results from the random centering of precipitation events causing floods. Any particular precipitation event may be centered over one watershed causing a major flood, but not over other watersheds in the region. Over the long term, the random centering of storms, should cause about the same distribution of floods for all the basins. Consequently, the recorded flood history from all the watersheds can be pooled or “regionalized” to effectively increase the record length for computing flood-frequency statistics.

A simple way of understanding how regionalization works is to consider a coin flipping process. Assume that individuals perform N trials of flipping a coin in M different watersheds. The N trials are not only independent of each other, but also independent of the trials performed in another watershed. The effective number of trials that can be used to estimate the probability of any outcome of flipping the coin, heads or tails, is MN.

Table 3.1 Inter-station correlation of annual peak flows with annual peaks at Sioux City, Missouri River

Station	from	to	*R-squared
Omaha, Mo Ri	1898	1997	.96
Nebraska City, Mo Ri	1898	1997	.90
St Joseph, Mo Ri	1898	1997	.75
Kansas City, Mo Ri	1898	1997	.49
Booneville, Mo Ri	1898	1997	.40
Hermann, Mo Ri	1898	1997	.30
Anoka, Miss Ri	1931	1995	.51
St Paul, Miss Ri	1898	1997	.42
Mankato, Minn Ri	1903	1995	.28
Winona, Miss Ri	1898	1995	.32
McGregor, Miss Ri	1937	1995	.30
Muscoda, Wisc Ri	1915	1976	.17
Dubuque, Miss Ri	1898	1996	.27
Clinton, Miss Ri	1898	1996	.36
Keokuk, Miss Ri	1898	1996	.25
Hannibal, Miss Ri	1898	1996	.20
Louisiana, Miss Ri	1928	1995	.27
Meredosia, Ill Ri	1921	1995	.18
Alton, Miss Ri	1928	1996	.19
St Louis, Miss Ri	1898	1996	.31
Chester, Miss Ri	1926	1996	.35

*Inter-station Correlation squared (coefficient of determination)

Table 3.2 Inter-station correlation of annual peak flows with annual peaks at Anoka, Mississippi River

Station	from	to	*R-squared
St Paul, Miss Ri	1931	1995	.88
Mankato, Minn Ri	1931	1995	.69
Winona, Miss Ri	1931	1995	.86
McGregor, Miss Ri	1937	1995	.77
Muscoda, Wisc Ri	1931	1976	.40
Dubuque, Miss Ri	1931	1995	.74
Clinton, Miss Ri	1931	1995	.72
Keokuk, Miss Ri	1931	1995	.51
Hannibal, Miss Ri	1931	1995	.44
Louisiana, Miss Ri	1931	1995	.44
Meredosia, Ill Ri	1931	1995	.24
Alton, Miss Ri	1931	1995	.42
St Louis, Miss Ri	1931	1995	.30
Chester, Miss Ri	1931	1995	.31

*Inter-station Correlation squared (coefficient of determination)

Increasing the effective record length for estimating flood-frequency statistics relies on the same principle as in the coin flipping example. In the most ideal circumstance, watershed response to precipitation would be equal for all the watersheds in the regional study. This is equivalent to assuming the occurrence of a storm is the same as the outcomes of coin tossing in the watersheds. The independence of the coin flipping trials is akin to assuming that the random centering of storms over the watershed results in the independent occurrence of annual peaks among the watersheds in the region selected. Ideally, the effective record length for the regional flood-frequency statistics would also be MN, where now N is the systematic record length at each of the M gages.

In practice, the regional observation of floods does not correspond perfectly to the coin tossing model. Watersheds have different responses to storm events. Furthermore, storms, do not in general occur randomly over individual watersheds, but may produce floods coincidentally over many watersheds. Consequently, peak annual flows are likely to be correlated to some degree. Correlation tends to reduce the effective record length obtained from a regional study. This can be appreciated, to some extent by considering the coin flipping example mentioned previously. If the coin tossing trials between the basin are correlated, the results of flipping a coin in a watershed may be influenced by the outcome in another watershed. In this case, the effective number of trials is no longer NM, but something less.

Of course, different methods exist for judging the worth of regional information given the deviations of the application situation from the ideal. Consider for example the application of the regional skew coefficient recommended in Bulletin 17B (IACWD, 1982).

Presumably, the regional skew for the study area would be obtained by the usual procedure of mapping the skew values and estimating iso-skew lines from this map. The squared deviation of the mapped skew values from these iso-lines are summed and divided by the number of values to obtain an estimate of the mean square error of the regional skew estimate. This estimate of the mean square error of the regional skew measures the lack of homogenous flood response and the influence of gage record length. The larger the mean square error, the less homogenous the response of the watersheds in the regional study to flood producing precipitation events. The improvement in skew estimates are obtained by weighting the regional and at-site skew values as follows (see page 5, IACWD, 1982):

$$G = \frac{mse_{G_r} G_s + mse_{G_s} G_r}{mse_{G_r} + mse_{G_s}} \tag{3.1}$$

where G is the skew adopted for computing the log-Pearson III distribution, G_r and G_s are respectively the regional and station skew values, and mse_{Gr} and mse_{Gs} are the corresponding mean square errors of estimation. Here, the mean square error of the station skew is a function of the record length, the greater the record the length, the smaller the mean square error.

Assume that the mse_{Gr} obtained for the Upper Mississippi study is of about the same as that obtained for the regional skew map provided in the guidelines of about 0.302. The mse_{Gs} can be determined from Table 1 (pg. 14) of the guidelines given an assumed record length and skew value. The record lengths shown in Table 2.1 for the study area main stem gages, are, for the most part, at least 100-years. Once the Corps finishes the unregulated flow estimation study, this will be true for all the stations used in the study. A preliminary study of the station skew for main stem gages resulted in values somewhat larger than the regional skew values obtained from the map in the Bulletin 17B guidelines of -0.3 to -0.4. For the purpose of this calculation, assumption of the map skew values is sufficient for demonstrating the influence of regional skew. Table 1 from the guidelines provides a mse_{Gs} estimate of 0.073 for G_s=-0.4 and a record length of 100-years. Consequently, the relative weight of regional and stations skews in computing the adopted skew from equation (3.1) is:

$$G = \frac{(0.302) G_s + (0.073)\ G_r}{(0.302) + (0.073)} = (0.81) G_s + (0.19) G_r \tag{3.2}$$

where it can be seen that the station skew will have significantly more weight than the regional skew values.

In conclusion, the high inter-station correlation and diverse response across the study area lead to the IAG recommending against the Bulletin 17B application of a regional skew value. Furthermore, a regional skew estimate is not likely to have much impact on the final estimates given the Bulletin 17B skew weighting procedure.

3.3 Estimation methods

3.3.1 Standard method of moments

The standard method of moments estimation procedure determines distribution parameters from the sample estimate of the product moments (see most standard texts on probability and statistics, e.g., Haan, 1977, for a discussion with applications to hydrology). The theory behind the method can be appreciated by a simple application of estimating the probability of obtaining a particular outcome of rolling a die. The probability can be estimated from the **first moment** or equivalently the **mean** outcome of the die tossing experiment. The mean outcome of the die rolling experiment is calculated as:

$$\mu_d = \frac{1}{6}[1+2+3+4+5+6] = \frac{21}{6} = 3.5 \qquad (3.3)$$

This equation can be written symbolically as:

$$\mu_d = P \sum_{i=1}^{i=6} d_i \qquad (3.4)$$

here P is the probability of an outcome of any role of the die (1/6), and d_i is the face value of each possible outcome, from i=1 to i=6.

To demonstrate the application of the standard method of moments, presume P is not known and needs to be estimated from observations. Clearly, P has to be (1/6) given the experiment involved, but determining P from observations is instructive concerning the method of moments.

An experiment is performed to obtain an average outcome by rolling the die 50 times to obtain:

$$\bar{d} = \left(\frac{1}{N}\right)\sum_{j=1}^{j=N} d_j^s = \left(\frac{1}{50}\right)\sum_{j=1}^{j=50} d_j^s = 3.2 \qquad (3.5)$$

where $\bar{d}$ is the average outcome from the N=50 observations, d_j^s . The average can be used as a **sample estimate** of the population mean. The key step in the standard method of moments is to equate the sample estimate of the mean obtained in (3.5) with the population estimate obtained from equation (3.4). Using the example, a sample estimate of the probability of obtaining any outcome from rolling a die is obtained by solving equation (3.4) for P, substituting the sample estimate of the mean for the population value, and using the known sum of outcomes, $\Sigma d_i=21$, to obtain:

$$P = \mu_d/(21) \approx \bar{d}/21 = 3.2/(21) = 0.1524 \tag{3.6}$$

The application of the standard method of moments to probability distributions is, in principle, the same as estimating the probability of obtaining a particular role of the die. However, the application to probability distributions is made to obtain parameters of the distributions rather than a probability; and, the magnitude and occurrence probabilities of stream flow are continuous unlike the discrete outcomes obtained from rolling a die.

The application equations for standard moment estimation to probability distributions are developed in an analogous manner to equations (3.4) and (3.5). For example, the mean or first moment for a continuous distribution is calculated as follows:

$$\mu_X \approx \sum_i X_i \Delta P_i \approx \sum_i X_i \Delta F_{X_i} \approx \sum_i X_i f_{X_i} \Delta X_i \rightarrow \int X f_X dX \tag{3.7}$$

where:

)P_i	=	the incremental probability that an observation of X will be in an interval [x_a < X # x_b] (this incremental probability corresponds to P in equation (3.4));
X_i	=	the flow or log-flow of the observed stream flows at the midpoint in the interval x_a and x_b (this value corresponds to d_i in equation (3.4));
F_{xi}	=	the cumulative distribution function (CDF) for the probability distribution, evaluated at X_i (this function gives the probability that the peak annual flow is less than X_i);
)F_{xi}	=	)P_i (the incremental probability is obtained as $F_X(x_b)$-$F_X(x_a)$, or the difference between the CDF evaluated at x_b and x_a;
f_{Xi}	=	the probability density function for F_X, evaluated at X_i, the density function has units of probability per flow and is obtained by differentiating F_X with respect to X;
)X	=	x_b - x_a (note that f_{Xi})X =)P_i);

and finally the integral shown is obtained in the limit as)X becomes infinitesimally small.

The distribution functions F_X or f_X have parameters that need to be estimated based on observed data. In general, the number of moments needed equals the number of distribution parameters. The general form for estimating additional **central moments** for the distribution is:

$$M_X^n = \int (X - \mu_x)^n f_X dX \tag{3.8}$$

where n is an exponent that corresponds to the nth moment M^n_X. Commonly, the second central moment is referred to as the **variance** and the third central moment divided by the **standard deviation** (the square root of the variance) cubed is referred to as the **skew coefficient**.

As an example application to a two parameter distribution, equations (3.7) and (3.8) for the Gumbel (Extreme Value Type I) distribution becomes:

$$\mu_X = \int_{X=-\infty}^{X=\infty} \frac{x}{\alpha}\left\{ e^{\left[\frac{-(x-\xi)}{\alpha} - e^{\frac{-(x-\xi)}{\alpha}}\right]}\right\} dx = \xi + 0.5772\alpha \qquad (3.9)$$

$$\sigma_x^2 = \int_{x=-\infty}^{X=\infty} \frac{(x-\mu_x)^2}{\alpha}\left\{ e^{\left[\frac{-(x-\xi)}{\alpha} - e^{\frac{-(x-\xi)}{\alpha}}\right]}\right\} dx = 1.645\alpha^2 \qquad (3.10)$$

where an analytic expression can be found between the population parameters □ α, ξ and the population mean and variance of the Gumbel distribution. The sample mean or average is substituted for μ_X and the sample variance for σ^2_X to obtain the parameters.

Typically, distributions used in flood frequency analysis have two parameters (e.g., Gumbel, log-Normal, and Gamma) or three parameters (log-Pearson type III, Generalized Extreme Value, Logistic and Pareto). There has been some applications with the four parameter Kappa and five parameter Wakeby distributions. Consequently, at most 5 moments need be calculated, although, typically only two or three are needed.

3.3.2 Bulletin 17B estimation

Bulletin 17B uses the method of moments as the basis for estimating parameters of the log-Pearson type III distribution. However, additional modifications to estimates described in detail in the bulletin. A recommended modification not used in this study is the application of historic information (i.e., estimates of discharge not part of the gage record but based on high water marks, newspaper accounts, etc.). Historic information was not used for three reasons. First, the information is not uniformly available throughout the study area, not well measured in some locations, and for runoff and channel conditions not relevant to current conditions. Second, the goal of the study is to evaluate the distributions by performing a comparison with observed frequencies of gage data. The length of record available is not necessarily relevant to the evaluation. Third and finally, historic information was not used in the evaluation study used to select the log-Pearson III distribution for Bulletin 17B from among competing distributions (see Thomas, 1985).

3.3.3 L-Moment estimation

Estimation with L-moments (or linear moments) has been popularized in combination with regional flood distribution estimation using index distributions (see Hosking and Wallis, 1997). L-moments have some potential advantages over estimation with standard moments. These advantage come from the linear nature of the L-moment estimators. L-moment estimators are a function of linear combinations of the ordered observations (thus the "L" designation). This linear property makes L-moment estimates of population moments nearly unbiased for moderate to large samples (period of record greater than 20-years) unlike standard moment estimates of the standard deviation and skew coefficient. Furthermore, the linear property results in the L-moment estimates being less sensitive or more robust to the occurrence of additional large observations in the record.

L-moments are applied in much the same way as standard moments. A relationship is developed between the distribution parameters and the population L-moments. Sample estimates of the L-moments are substituted for the population values to obtain the distribution parameters.

Calculation of the L-moments is most conveniently done by utilizing these moments relationship with the following particular form of probability weighted moments (PWMs):

$$\beta_r = \int X(1\text{-}F_X)^r f_X dX \quad r=0,1,2,\text{ etc.} \tag{3.11}$$

Notice that the PWM computed in this equation are linear in the flow values; X; whereas, the standard moments are proportional to X raised to a power in equation (3.8). L-moments can be computed from a linear combination of the PWMs. For example, the first four L-moments can be computed as:

$$\lambda_1 = \beta_0 \tag{3.12}$$

$$\lambda_2 = 2\beta_1 - \beta_0 \tag{3.13}$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 \tag{3.14}$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \tag{3.15}$$

The L-moments are linear in the observations X , since these moments are linear in the PWMs.

Sample estimates of the L-moments can be computed by substituting the sample estimates of the PWMs into the above equations. Sample estimates of the PWM can be obtained by adding linear combinations of the ranked observations, $X_{1:N} \le X_{2:N} \ldots\ldots \le X_{j:N} \ldots\ldots X_{N:N}$, where $X_{1:N}$ is the smallest, and $X_{N:N}$ is the largest of the N observations of stream flow. These $X_{j:N}$ are referred to as order statistics. As an example of the computation of the sample moments, consider the first two PWMs (see Hosking and Wallis, 1997 for further details):

$$b_0 = \frac{1}{N}\sum_{j=1}^{j=N} X_{j:N} \qquad (3.16)$$

$$b_1 = \frac{1}{N}\sum_{j=2}^{j=N} \frac{(j\text{-}1)}{(N\text{-}1)} X_{j:N} \qquad (3.17)$$

Examination of equation (3.16) reveals that b_0 is the sample mean or average.

There is in general an analogy between L-moments and standard moments. For example, the first L-moment and the first standard moment are the mean, the standard moment variance and the second L-moment measure the spread of the distribution, and the third L-moment and the standard moment skew measure the asymmetry of the distribution.

As in the case of the standard moments (see equations (3.9) and (3.10)) a relationship between the L-moments and distribution parameters needs to be developed. See Hosking and Wallis, 1997, for details on the development of these equations.

3.3.4 Regression with Censored Data Sets

Applications of regression analysis to obtain the parameters of a probability distribution is a recognized technique (Beard, 1962 , and Kroll and Stedinger, 1996), although not widely applied. The method will be applied to censored portions of the data, where the censoring level can be based on either a particular minimum flow level or exceedance probability. Channel capacity or a median flow level might be used as censoring levels.

Application of the regression to the censored data is performed in the spirit of the low-outlier censoring performed in Bulletin 17B or any other attempts to fit only the tail of the distribution. Basically, this estimation approach has the advantage of not being unduly influenced by the possibly small censored flood values.. The censored flow values have some influence by providing information on the plotting position of the observed largest flows.

Parameters are estimated from the censored data by postulating the following linear relationship:

$$X_{j:N} = a + bK_{j:N} + e_{j:N} \qquad j \ \geq \ j_c \qquad (3.18)$$

where $X_{j:N}$ is the jth ordered observation, $K_{j:N}$ is a dimensionless measure of the flow for an exceedance probability corresponding to rank j and the distribution of interest, a and b are distribution parameters, $e_{j:N}$ is the regression residual for the jth ordered value and j_c is the plotting position corresponding to the censoring level. The parameters a and b can be computed easily using standard regression methods once $K_{j:N}$ is computed.

The computation of $K_{j:N}$ depends on the distribution involved. The computation is straightforward in the case of some two parameter distributions. Three parameter distributions would require an additional algorithm to optimize estimates of a third parameter or a non-linear regression scheme would need to be instituted. A two parameter distribution should be sufficient to describe the censored portion of the data, and, thus be parsimonious. Consequently, to avoid the complications resulting from a third parameter and by presuming that two parameter distribution is sufficient, three parameter distributions were not investigated as part of the regression analysis at this time.

Parameters for the two-parameter log-Normal and Gumbel distribution were computed using the regression scheme. The regression equation for the log-Normal distribution is given by:

$$X_{j:N} = \overline{X} + K_{j:N} S + e_{j:N} \qquad (3.19)$$

where is the sample mean of the logarithms and S is the sample standard deviation. The standard normal deviate corresponds to the deviate for a normal distribution, with mean zero and standard deviation equal to one, given a particular exceedance probability. The exceedance probability is computed from a plotting position formula given j and N.

A similar relationship can be derived for the Gumbel distribution by examining this distribution's CDF:

$$X = \xi + \alpha\left[-\log\left(-\log\left(P\right)\right)\right] \qquad (3.20)$$

where ξ and α are parameters. A linear relationship is obtained if $K_{j:N} = -\log(-\log(P))$. Application to equation (3.20) is possible given that P is computed from some suitable plotting position formula.

3.4 Distributions

Distributions selected for testing corresponded to the standard two and three parameter ones described in the literature (see Table 3.3). Additionally, the five parameter Wakeby distribution was selected because it has been often applied in combination with L-moment estimation procedures.

Table 3.3: Distributions used in comparisons

Distribution	Cumulative distribution or density function[2]	transform[3]	parameters
Gumbel (GEV k=0)	$y=-\log(-\log(F))$	$y=\frac{x-\xi}{\alpha}$	∀, >
Generalized Extreme Value (GEV)	$y=1-(-\log(F))^k/k \quad k \neq 0$	$y=\frac{x-\xi}{\alpha}$	∀, >, k
Generalized Pareto	$y=1-(1-F)^k \quad k \neq 0$ $y=-\log(1-F) \quad k=0$	$y=\frac{x-\xi}{\alpha}$	∀, >, k

Generalized Logistic	$y=1-\{(1-F)/F\}^{k} \quad k \neq 0$ $y=\log\{(1-F)/F\} \quad k=0$	$y=\dfrac{x-\xi}{\alpha}$	∀, >, k
log-Normal	$f(y)=\dfrac{1}{\sqrt{2\pi}}e^{-y^2/2}$	$y=\dfrac{\log(x)-\mu}{\sigma}$	μ,Φ
Gamma[1]	$f(y)=\dfrac{y^{\alpha-1}e^{-y/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$	y=x	□,□
log-Pearson III	$f(y)=\dfrac{y^{\alpha-1}e^{-y/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$	$y=\dfrac{\log(x)-\xi}{\beta}$	□,□,□
Wakeby	$x=\xi+\dfrac{\alpha}{\beta}\{1-(1-F)^{\beta}\}-\dfrac{\gamma}{\delta}\{1-(1-F)^{-\delta}\}$	---------------	□,□,□,□,□

[1]э( ) is the gamma function
[2]F is the cumulative probability (non-exceedance probability), f(y) is probability density function
[3]x is the quantile or flow value of interest

4. Comparison study

4.1 Comparison methodology

The distributions and estimation procedures were evaluated using two different criteria. The first criterion investigated how well the distribution fit the entire data set (full period of record). Basically, this judges the "curve fitting" capability of the combined distribution and estimation procedure. A second criterion employing split sample testing was selected because of its use in the original investigation to select the methods in Bulletin 17B. In split sample testing the distribution is estimated from half of the data and then evaluated based on a comparison with plotting positions for the remaining half of the data. The divided data sets were obtained by examining: 1) a **forecast** (estimate parameters for first half and compare predictions to second half); 2) a **hindcast** (estimate parameters for second half and compare predictions to first half); 3) **an alternating odd sequence** (estimate parameters from earliest first, third, etc. values and compare to remaining data); and finally, 4) **an alternating even sequence** (estimate parameters from earliest second, fourth, etc. values and compare to remaining data). The different methods for dividing the data sets were selected to ascertain if trends in the flow series influence the comparison results.

Comparisons were made for predicted flood quantiles and exceedances. The comparison of exceedances was done in the original studies to establish Bulletin 17B (see Appendix14, IACWD, 1982). However, some approximations were needed to correct estimators for bias in comparing the various methods tested. Consequently, a comparison of quantile predictions was also made to attempt to avoid, or at least obtain a different perspective of estimator bias.

4.2 Quantile comparisons

4.2.1 Error measures

The error measures used in comparing quantiles were average bias, average absolute relative error and mean square error. The bias and mean square error are computed using standardized deviates, which are computed as:

$$K_d = \frac{\left(Q_{d,P} - \overline{Q}\right)}{S} \qquad (4.1)$$

$$K_i = \frac{\left(Q_{i,P} - \overline{Q}\right)}{S} \qquad (4.2)$$

where is the sample mean, and S, the sample standard deviation of the flows at a particular gage, $Q_{d,P}$ is the distribution predicted flow, and $Q_{i,P}$ the interpolated flow from plotting positions, at exceedance probability P.

The computation of the distribution predicted flow values, $Q_{d,P}$, was accomplished using a variety of different programs and statistical libraries. Computer program FFA (HEC, 1992) was used to implement the Bulletin 17b procedures to obtain the log-Pearson III distribution. The application of the standard moments and regression to estimate parameters for other distributions used in the analysis was performed using software specifically developed for this project. Application of L-moments was accomplished using software provided by Hosking (1996).

Different plotting position formula were used to test the sensitivity of the computed differences to the estimated values of $Q_{i,P}$. The plotting position formulas used are:

$$P = \frac{j-0.4}{N+0.2} \quad \text{Cunnane} \tag{4.3}$$

$$P = \frac{j}{N+1} \quad \text{Weibull} \tag{4.4}$$

$$P = \frac{j-0.35}{N} \quad \text{Hosking and Wallis} \tag{4.5}$$

Cunnane's plotting position formula is probably the most appropriate in that it is reported to be nearly unbiased with regard to flows for most distribution used in hydrologic applications (see Stedinger et al., 1992). The Weibull (see Stedinger et al., 1992) plotting position formula, being unbiased with regard to estimating probability, is probably least well suited for flow comparisons. However, this plotting position provides significantly different estimates than Cunnane's and; consequently, is valuable for testing the sensitivity of study results to the plotting position selection. The plotting position formula proposed by Hosking and Wallis (1997) is useful for computing sample L-moments for certain distributions, such as the GEV and was used to provide additional information on the sensitivity of results to formula selection.

The interpolation of $Q_{i,P}$ from plotting positions was a function of the distribution being used. Effectively, the interpolation scheme involved a transformation to linear (i.e., as if probability paper was created for each distribution). The interpolated flow values, $Q_{i,P}$ were obtained from the observed flow values transformed to the same scale. As it turned out the difference between interpolated values for different distributions was not significant to the evaluation procedure.

The bias corresponding to a particular distribution and estimation method is computed as the average over all gages of the difference between predicted and observed flow deviate values at a particular exceedance probability. The absolute relative error of a prediction for a particular gage location is given by subtracting the deviates in equations (4.1) and (4.2), and dividing by the average:

$$\text{absolute relative error} = \frac{\left|\left(Q_{i,p} - Q_{d,p}\right)\right|}{0.5\left(Q_{i,p} + Q_{d,p}\right)} \tag{4.6}$$

and the squared error is obtain by squaring the difference of the deviates in equations (4.1) and (4.2):

$$\text{squared error} = \left(\frac{Q_{i,p} - Q_{d,p}}{S} \right)^2 \qquad (4.7)$$

As in the case of the bias, the absolute relative error and squared error are summed for all the gages for a given exceedance probability for use in evaluating the distribution and estimation procedure combination.

In summary, three different error measures were used to evaluate the distributions. These error measures were used in both the evaluation of the distribution and estimation methods ability to adequately describe the full period of record and in split sample testing.

4.2.2 Quantile comparison results

The results of the analysis are separated into the analysis of the unregulated data estimated by the Corps of Engineers for the main stem Upper Mississippi River and tributaries, and the data obtained for the smaller watersheds obtained from the USGS data base.

The following abbreviations are used in the tables to signify the combined distribution and estimation pairs:

l-gamma	Gamma distribution with L-moments
l-GEV	Generalized Extreme Value distribution with L-moments
l-g.logist	Generalized Logistic distribution with L-moments
l-g.pareto	Generalize Pareto distribution with L-moments
l-gumbel	Gumbel distribution with L-moments
l-l.normal	log-Normal distribution with L-moments
l-wakeby	Wakeby distribution with L-moments
m-lpIII	log-Pearson III with Bulletin 17B procedure
m-l.normal	log-Normal distribution with standard moments
m-gumbel	Gumbel distribution with standard moments
c-l.normal	log-Normal with regression applied to data censored below50% chance exceedance
c-gumbel	Gumbel with regression applied to data censored below 50% chance exceedance

The raw data used to compute the measures of error, estimates of distribution parameters, and the distribution comparison errors for each gage are provided in text files on a floppy disk accompanying this report . Please see the README.TXT file for a description of the files. A complete description of the tabular results for all plotting position formula tested is provided in Appendix A. Results presented in this section are typical of the complete set of comparisons. Appendix B provides selected plots comparing distributions and the plotted data.

Notice that comparison results with plotting positions estimates shown in the tables are not available for all exceedance probabilities depending on the test performed. This occurs because the combination of record length and plotting position formula chosen may preclude the estimation of the flow for that exceedance probability at a particular station. A minimum of five stations was required to perform the comparisons.

The ultimate goal of the comparisons shown in the tables is to determine if the distributions selected as "best" based on the selection criteria provide significantly different estimates of flood frequencies in the range of interest than the standard estimates obtained using the Bulletin 17B Guidelines (IACWD, 1982). Examining the comparison results with Cunnane's plotting positions for the unregulated data in Tables 4.1-4.3 reveals no preferred distribution. The distribution selected depends to a great extent on the type of test performed (period of record, forecast, hindcast, etc.) and the exceedance probability. Some overall preference is revealed irrespective of the selection criteria (bias, relative error, and mean square error). In general, these preferences are: 1) standard moments application with the log-normal distribution for the forecast test; 2) the L-moment application with the generalized logistic at the 10% chance exceedance; and 3) the generalize pareto at the 2% chance exceedance for the hindcast test. General preference, but with some exception, was demonstrated for the L-moment application with the Wakeby distribution in period of record testing; and for the L-moment application with the generalized logistic and the standard moments application to the log-normal distribution in the alternate even testing.

The selection results did not differ greatly when comparison were made to plotting positions obtained from either the Weibull or Hosking and Wallis formulas as is shown in Appendix A. The only consistent preference shown in the testing was for the application to the log-normal distribution to forecast split-sample testing.

The distribution comparisons for the USGS data were similar to the unregulated data in that no clear preference was demonstrated as is shown in Tables 4.4-4.6. In general, the standard moments application was preferred for the forecast test, except for the comparisons with Cunnane's plotting position, mean square error selection criterion at the 2% chance exceedance probability (Table 4.6). Comparisons using other plotting position formula did not reveal a preferred distribution as is shown in Appendix A.

Table 4.1: Distributions with minimum bias for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*c-l.normal	*c-l.normal	*l-g.logist
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*m-l.normal	**l-gumbel	-----
alternate even split	*l-g.logist	**m-l.pIII	-----

*Based on 23 Stations
**Based on 22 Stations

Table 4.2: Distributions with minimum absolute relative error for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*l-wakeby	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-wakeby	**l-gamma	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*Based on 23 Stations
**Based on 22 Stations

Table 4.3: Distributions with minimum mean square error for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*l-wakeby	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-l.normal	**l-gumbel	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*Based on 23 Stations
**Based on 22 Stations

Table 4.4: Distributions with minimum bias for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
[*]Period of record	[*]c-gumbel	[*]l-wakeby	[*]m-l.pIII
[**]forecast split record	[**]m-l.normal	[**]m-l.normal	-----
[**]hindcast split record	[**]l-g.logist	[**]l-gumbel	-----
[**]alternate odd split	[**]m-l.normal	[**]c-l.normal	-----
[**]alternate even split	[**]l-g.logist	[**]c-l.normal	-----

[*]Based on 11 stations
[**]Based on 8 stations
[1]Exceedance probability

Table 4.5: Distributions with minimum relative error for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	[*]c-gumbel	[*]l-wakeby	[*]l-g.logist
Forecast split record	[*]m-l.normal	[**]m-l.normal	-----
hindcast split record	[*]l-g.logist	[**]l-gumbel	-----
alternate odd split	[*]m-l.normal	[**]l-gamma	-----
alternate even split	[*]l-g.logist	[**]l-gumbel	-----

[*]Based on 11 stations
[**]Based on 8 stations
[1]Exceedance probability

Table 4.6: Distributions with minimum mean square error for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	[*]l-gamma	[*]l-wakeby	[*]l-wakeby
Forecast split record	[*]m-l.normal	[**]l-l.normal	-----
hindcast split record	[*]l-g.logist	[**]l-gumbel	-----
alternate odd split	[*]l-l.normal	[**]c-l.normal	-----
alternate even split	[*]l-g.logist	[**]l-gumbel	-----

[*]11 stations
[**]8 stations
[1]Exceedance probability

Given no clear preference, the difference between best “distribution” and the standard estimates obtained with the log-Pearson III was examined for different test and estimation combinations. Table 4.7

Table: 4.7 Comparison of average %difference between log-Pearson III and best distribution/ estimation procedure combination for the 1% chance event, best distribution has minimum mean square error @2% chance exceedance, Cunnane plotting position formula

test type	(1)	(2)	(3)	(4)	(5)	(6)
Period of record	l-wakeby*	.31	m-gumbel	2.11	c-gumbel	1.76
forecast split record	l-l.normal	8.67	m-l.normal*	9.94	c-l.normal	.80
hindcast split record	l-g.pareto*	-7.32	m-gumbel	2.11	c-l.normal	.80
alternate odd split	l-gumbel*	2.39	m-gumbel	2.11	c-l.normal	.80
alternate even split	l-l.normal	8.67	m-l.normal*	9.94	c-l.normal	.80

(1)Best distribution based on mean square error, L-moment estimation

(2)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

(3)Best distribution based on mean square error, standard moment estimation

(4)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

(5)Best distribution based on mean square error, regression application to uncensored data

(6)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

shows the average difference between the log-Pearson III and the “best” distributions for each estimation method (standard moments, L-moments, and regression) obtained from the mean square error criterion applied for the 2%. Note that these are not the best distribution/estimation pairings as shown in Table 4.3, but the distribution for each estimation technique having minimum mean square error in comparison to the plotted data. In the comparisons shown in Table 4.7, estimates of the 1% chance event were made based on the full period of record. As can be seen, the log-normal distribution estimated with either standard moment or L-moments predicts significantly greater 1% chance events; whereas, the generalized Pareto distribution estimated with L-moments predicts significantly lower values. These results reflect the same magnitude of differences that would be found with other distributions and estimation methods. In general, the generalized Pareto distribution estimated with L-moments seemed to be more negatively skewed than the log-Pearson III, thus producing lower estimates of the infrequent floods. In contrast, the log-Normal distribution estimated with either standard or L-moments, provides greater estimates given the distribution’s greater (zero) skew than the negative skew obtained for the log-Pearson III. The other distribution-estimation method combinations provide comparable predictions on the average with the log-Pearson III distribution.

A concern that arises from these comparisons is the preference shown for the log-Normal distribution-standard moment combination for the forecast test. This preference is evidence for some positive trend, or at least greater frequency of flooding in the latter half of the period of record given the greater predictions of the 1% chance event obtained with the log-Normal distribution as shown in Table 4.8. This trend can be appreciated by examining Figure 4.1, which compares estimates of the 1% chance flood obtained from estimating the log-Pearson III distribution estimated from the first and second half of the period of record at the gage locations studied. This result provides some initial rationale for investigating if there is some trend towards larger floods in the period of record. Section 8 describes the

Table 4.8: %Difference between log-Pearson III and log-normal distribution for 1% chance event

Location	m-l.pIII[1]	l-l.normal[2]	m-l.normal[3]	c-l.normal[4]
Sioux City, Mo Ri	345100	16.23	19.10	.87
Omaha, Mo Ri	334700	14.05	16.38	.61
Nebraska City, Mo Ri	382300	15.89	16.99	.76
St Joseph, Mo Ri	384600	.13	1.76	-3.02
Kansas City, Mo Ri	559200	-1.62	.38	2.98
Booneville, Mo Ri	673600	3.27	3.45	2.74
Hermann, Mo Ri	894400	-.03	-1.11	-1.30
Anoka, Miss Ri	80850	20.84	22.89	1.91
St Paul, Miss Ri	141500	11.68	12.21	1.64
Mankato, Minn Ri	77620	34.42	40.80	13.56
Winona, Miss Ri	230300	4.47	3.66	.17
McGregor, Miss Ri	252500	.69	1.11	5.60
Muscoda, Wisc Ri	85580	24.76	27.50	5.08
Dubuque, Miss Ri	261900	13.78	17.89	2.91
Clinton, Miss Ri	278800	9.78	10.27	.69
Keokuk, Miss Ri	360400	12.81	14.60	3.82
Hannibal, Miss Ri	435600	16.89	23.06	3.31
Louisiana, Miss Ri	498400	4.26	3.19	.88
Meredosia, Ill Ri	136900	18.47	19.59	.88
Alton, Miss Ri	604800	2.83	2.31	-1.68
St Louis, Miss Ri	1055000	8.49	8.83	.10
Chester, Miss Ri	1180000	16.57	17.33	-.17
Thebes, Miss Ri	1164000	17.93	19.20	.69
average		11.59	13.10	1.87

[1]Prediction of 1% chance event using LP III distribution
[2]Percent difference prediction log-Normal estimated with L-moments
[3]Percent difference prediction log-Normal estimated with standard moments
[4]Percent difference prediction log-Normal estimated using censored data

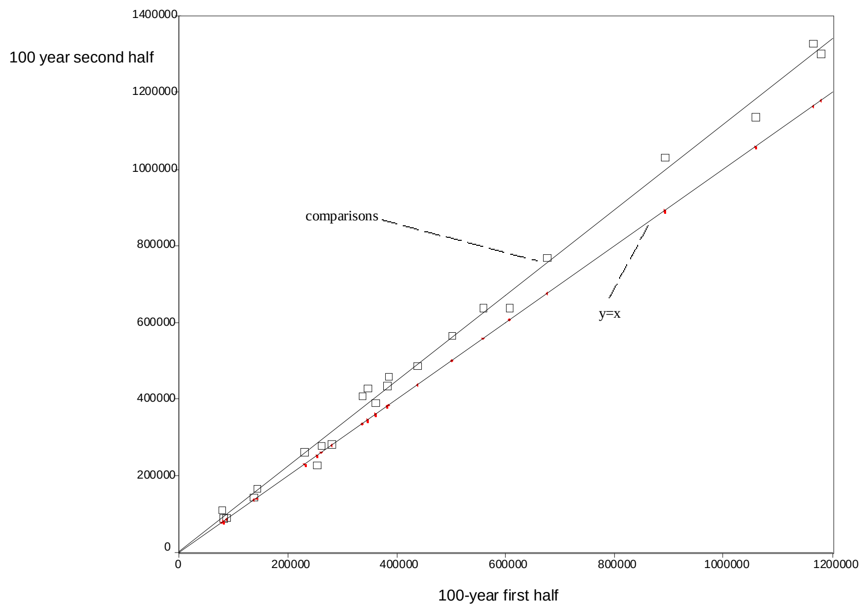

Figure 4.1: Comparison of 100-year flood estimates (cfs), first and last half of the record log-Pearson III distribution

application of standard statistical tests to determine if the apparent increase in flooding is due to some type of non-randomness or non-stationarity.

In conclusion, the distribution selection procedure did not reveal any clear preference for a particular distribution/estimation method combination. The forecast split-sample testing did result in selection of the log-Normal distribution/standard estimation procedure. This result is of some concern since this combination results in significantly higher prediction of the 1% chance event than would be obtained with the Bulletin 17B applications with the log-Pearson III distribution. This forecast test selection may provide some evidence for non-randomness in the period of record. The period of record will be analyzed for this possibility in Section 8. Comparison of the log-Pearson III with other distributions revealed that: 1) the generalized Pareto/L-moment combination is more negatively skewed than the log-Pearson III in this application and provides smaller estimates of the 1% chance flood; and 2) the other distribution-estimation method combinations (except for the log-normal standard moment or L-moment combinations) provide very comparable estimates of the 1% chance flood. Consequently, greater values of the 1% chance flood event will not be obtained on the average unless the log-normal distribution/standard moment combination is selected based on the forecast split sample testing.

4.2.3 Application to Missouri River Basin

The study area is very large, encompassing a variety of climatic regions and topographic features. Expecting a single distribution to describe the flood frequencies in the entire area may not be realistic. However, the goal of this study is to select a distribution that is appropriate for very large drainage areas. Unfortunately, this requires that gages from a large cross-section of climatic regions be chosen to obtain a significant number of gages for a distribution selection study.

In this section, the influence of region selection is investigated to some extent by using gages on the Missouri River main stem for distribution selection. This reduces the number of gages to seven, but provides some information on the influence of region. The same testing procedures, estimation procedures and distributions were examined as in the previous section.

The results of the selection analysis shown in Tables 4.9-4.11 are similar to those found previously for all the stations in the unregulated data set and USGS data. As before, the normal distribution was selected for the forecast test, primarily in combination with standard moment estimation. The distributions selected for other tests was again mixed, but with the log-normal distribution preferred more often than in the previous analyses. This preference for the log-normal distribution may be explained by the somewhat higher skew coefficient values obtained for the stations on the Missouri main stem (see figure 7.1). Although, these results are not that different, the differences do provide some reasons for considering distributions for sub-regions of the study. Studies might be considered that examine mixed population analysis where snow melt is important, or consider the influence of channel changes where sediment movement is a factor.

Table 4.9: Distributions with minimum bias for selected exceedance probabilities, Cunnane plotting position, Missouri River main stem stations

*Test	[1]0.100	[1]0.020	[1]0.010
Period of record	l-wakeby	m-l.normal	m-l.normal
forecast split record	m-l.normal	m-l.normal	-----
hindcast split record	l-wakeby	l-gamma	-----
alternate odd split	c-gumbel	c-gumbel	-----
alternate even split	l-wakeby	m-l.normal	-----

*Based on 7 stations
[1]Exceedance probability

Table 4.10: Distributions with minimum relative error for selected exceedance probabilities, Cunnane plotting position, Missouri River main stem stations

*Test	[1]0.100	[1]0.020	[1]0.010
Period of record	l-wakeby	c-gumbel	m-l.normal
forecast split record	m-l.normal	m-l.normal	-----
hindcast split record	l-wakeby	l-g.pareto	-----
alternate odd split	l-l.normal	l-gumbel	-----
alternate even split	l-g.logist	m-l.normal	-----

*Based on 7 stations
[1]Exceedance probability

Table 4.11: Distributions with minimum mean square error for selected exceedance probabilities, Cunnane plotting position, Missouri River main stem stations

*Test	[1]0.100	[1]0.020	[1]0.010
Period of record	l-wakeby	c-gumbel	m-l.normal
forecast split record	l-l.normal	m-l.normal	-----
hindcast split record	l-wakeby	l-g.pareto	-----
alternate odd split	l-l.normal	l-l.normal	-----
alternate even split	l-g.logist	m-l.normal	-----

*Based on 7 stations
[1]Exceedance probability

4.3 Exceedances

4.3.1 Error measures

Comparisons were made between exceedance probabilities predicted by a distribution and the Weibull plotting position formula. The Weibull formula was selected because it provides an unbiased estimate of exceedance probability for a given ranked observation. As in the case of the quantile comparisons, the distribution is estimated from half of the record obtained in the split sample procedure. The remaining half of the record in the split sample is ordered and plotting positions obtained. The distribution is used to estimate an exceedance probability of flow values at selected rankings of the ordered observations. The comparison is made between this estimated exceedance probability and the estimate obtained from the plotting position. The top-ranked, 10^{th} ranked and median values ranks of the ordered observations selected in this comparison study were also used in the in studies leading to the selection of methods for the Bulletin 17B (see appendix 14, IACWD 1982).

The measures of accuracy used in comparing exceedances are as follows:

1. Bias, estimated as the sum of the difference between distribution predictions and plotting position values for all gages tested;
2. Mean square error, the average sum of squared differences between predicted exceedance and estimated plotting position values over all the gages;
3. Mean relative error, equal to the average of the difference $(1 - (\text{plotting postion/prediction})^2)$ for all gages tested.

Bias was also investigated as an aggregate value for all stations at a particular exceedance probability. This aggregate bias is computed by comparing: 1) the number of exceedances of a particular quantile counted in half of the record; to, 2) the predicted number of exceedances of the quantile predicted by the distribution estimated from the remaining half of the data in the split sample test. For example, the number of observed exceedances of the 1% chance quantile in a 1000 years of station record is compared to the expected value of 10.

These measures of accuracy encompass the comparisons made for Bulletin 17B except bias was substituted for prediction variance. Prediction variance could not be computed in this effort because of the lack of a sufficient number of gages to delineate regions for comparisons.

The importance of bias was investigated by using expected probability estimates (see appendix 11, IACWD 1982) in the comparison study for the log-normal and log-Pearson III distribution. No such theoretical expected probability estimators exist for the other distribution/estimation pairings used in this comparison study. Empirical expected probability estimates were made for other distribution used in studies leading to Bulletin 17B study. However, this was not felt to be desirable or necessary in this study. The goal of these comparisons was to ascertain if methods would be superior to the Bulletin 17B method. Consequently, a direct comparison was made between other recommended procedures and the log-Pearson III estimated using expected probability.

4.3.2 Exceedance comparison results

The comparison with observed exceedances did not produce superior results to that of the quantile comparison because a clearly superior distribution/estimation method pairing was not identified. A summary of results presented subsequently provides a mixed picture of the relative performance of the different distribution/estimation pairings. See appendix A for complete tabular comparisons made in the study.

The comparison of aggregate bias shown in Table 4.12 for the 1.0% chance quantile is indicative of the differences between the different distribution/estimation pairings for the 10.0%, and 2.0% comparisons. The log-normal/standard moments pairing performed best at the 1% level as shown, and also did best at the 10% level; but, the Gumbel/L-moment comparisons did best for exceedances of the 2.0% quantile.

The comparison of estimated exceedance probabilities was performed for both the full data set (see Table 2.1) and for a consistent period of record (1933-1996). A consistent period of record was considered to obtain comparisons for plotting positions estimates based on the same record length; and consequently, the same estimation accuracy (the error in estimating the exceedance probability of the top ranked quantile depends on the record length).

The results of both the full record comparisons summarized in Tables 4.13-4.15 and for the consistent period of record in Tables 4.16-4.18 do not result in any clear choice. L-moment estimation techniques tend to dominate, although the log-Normal/standard moment pairing is selected often.

In conclusion, the split sample testing with exceedances was not more revealing than using quantiles. L-moment estimation seems preferred, but the log-normal distribution (method of moments or L-moments) is also chosen in a significant number of cases. Application of expected probability does not result in any great advantage to the method of moments application with the log-normal or log-Pearson III distributions.

Table 4.12: Estimated bias for distribution/estimation pairings, 0.010 exceedance probability

distribution	observed	proportion	ratio
l-gamma	20	.0185	1.85
l-GEV	29	.0268	2.68
l-g.logistic	19	.0175	1.75
l-g.pareto	53	.0489	4.89
l-gumbel	16	.0148	1.48
l-l.normal	14	.0129	1.29
l-wakeby	31	.0286	2.86
m-l.pIII	18	.0166	1.66
m-l.normal	12	.0111	1.11
m-gumbel	17	.0157	1.57
c-l.normal	18	.0166	1.66

[1]Observed exceedances of 0.01 discharge from 1084 total observation
[2]Observed exceedances divided by total observations
[3]Proportion/0.01

Table 4.13: Best distribution/estimation pairing selected based on bias, full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-l.normal	l-gumbel	l-GEV
forecast split record	m-l.normal	l-g.pareto	c-gumbel
hindcast split record	l-g.logist	l-wakeby	l-g.pareto
alternate odd split	l-g.pareto	l-g.pareto	c-l.normal
alternate even split	l-g.logist	l-g.logist	l-l.normal

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10th ranked exceedance probability event in reserved period of record
[3]Comparison with median ranked event exceedance probability in reserved period of record

Table 4.14: Best distribution/estimation pairing selected based on mean square error, full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-g.logist	l-wakeby	l-g.pareto
forecast split record	l-l.normal	m-l.normal	l-g.pareto
hindcast split record	l-g.logist	l-l.normal	l-g.pareto
alternate odd split	l-g.logist	m-l.normal	l-g.pareto
alternate even split	l-g.logist	m-l.normal	l-g.pareto

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10th ranked event exceedance probability in reserved period of record
[3]Comparison with median ranked event exceedance probability in reserved period of record

Table 4.15: Best distribution/estimation pairing selected based on average of (one - relative error squared), full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-g.logist	l-wakeby	l-g.pareto
forecast split record	m-l.normal	m-l.normal	l-g.pareto
hindcast split record	l-g.logist	l-l.normal	l-g.pareto
alternate odd split	l-g.logist	l-l.normal	l-g.pareto
alternate even split	l-g.logist	m-l.normal	l-g.pareto

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10th ranked event in exceedance probability reserved period of record
[3]Comparison with median ranked event exceedance probability in reserved period of record

Table 4.16: Best distribution/estimation pairing selected based on bias, 1933-1996 period of record

Comparison	Top ranked	10^{th} ranked	Median
Period of record	m-l.normal	m-gumbel	l-g.logist
forecast split record	c-gumbel	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	m-l.normal	l-g.pareto
alternate odd split	l-g.logist	m-l.normal	l-g.pareto
alternate even split	m-l.normal	l-g.pareto	l-wakeby

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10^{th} ranked event exceedance probability in reserved period of record
[3]Comparison with median ranked even exceedance probability t in reserved period of record

Table 4.17: Best distribution/estimation pairing selected based on mean square error, 1933-1996 period of record

Comparison	Top ranked	10^{th} ranked	Median
Period of record	l-g.logist	l-wakeby	l-wakeby
forecast split record	m-l.normal	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	l-g.pareto	l-g.pareto
alternate odd split	l-g.logist	m-l.pIII	l-g.pareto
alternate even split	l-wakeby	m-l.normal	l-g.pareto

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10^{th} ranked even exceedance probability t in reserved period of record
[3]Comparison with median ranked event exceedance probability in reserved period of record

Table 4.18: Best distribution/estimation pairing selected based on average of (one - relative error squared), full period of record

Comparison	Top ranked	10^{th} ranked	Median
Period of record	l-g.logist	l-wakeby	l-wakeby
forecast split record	m-l.normal	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	l-g.pareto	l-g.pareto
alternate odd split	l-g.logist	m-l.pIII	l-g.pareto
alternate even split	l-wakeby	m-l.normal	l-g.pareto

[1]Comparison with top ranked event exceedance probability in reserved period of record
[2]Comparison with 10^{th} ranked event in exceedance probability reserved period of record
[3]Comparison with median ranked event exceedance probability in reserved period of record

5. Sensitivity analysis methods

5.1 Introduction

The TAG recommended a sensitivity analysis given the inability of the comparative study to identify a best distribution/estimation method pairing. The recommended sensitivity analysis compared the Bulletin 17B methodology with both at-site techniques and regional shape estimation. Regional shape estimation (Lettenmaier and Potter, 1985, Lettenmaier et al., 1987, Stedinger and Lu (1995), Hosking and Wallis, 1997 chapter 8, pg. 148) involves using at-site estimates of distribution location and scale parameters and substituting a regional shape parameter for the at-site estimate. For example, the log-Pearson III distribution would be computed using the at-site sample mean and standard deviation, but a regional skew would be substituted for the at-site skew estimate. This represents a departure from the earlier IAG recommendations against using regional information.

A preliminary analysis of the worth of regional information was performed by TAG member Hosking (1998) of the index flood /L-moment (see Hosking and Wallis, 1997); and, 2) regional shape estimation techniques for a group of stations within the project study area. He concluded from the results of analyzing the study area gages that (pg. 1):

> *..... because long records are available and the frequency distributions are not particularly heavily tailed, at-site estimation is already fairly reliable and there is not room for improvement by regional analysis.*

However, the TAG still felt it was worthwhile to consider regional shape estimation after reviewing Hosking's results which demonstrated some advantage to this approach; and, because past research into this approach demonstrated its value. The recommendation was to relate a regional shape parameter (e.g., the skew coefficient) to drainage area using generalized least squares. The regional shape parameter is then used in place of the at-site estimate to obtain the distribution.

The procedure for performing the sensitivity analysis recommended by the Technical Advisory Group (TAG) involves selecting a candidate distribution, obtaining regional skew and shape estimates, and making comparisons with flood distribution estimates computed via the Bulletin 17B guidelines. Complementary analyses are used to select a candidate distribution and to obtain estimates using the regional shape procedure recommended by the TAG as is described in section 5.2. Section 5.3 describes the expected moments algorithm method recommend by the TAG as an at-site estimation procedure additional to the estimation procedures already described section 3 for application of regression estimation to censored data sets.

5.2 Regional shape estimation and selection of a candidate distribution

5.2.1 Determining the region and selecting the distribution

The approach taken to estimate a regional shape parameter and select a candidate distribution was based on the procedure developed by Hosking and Wallis (1997) for regional flood distribution estimation. In this instance, the focus is only on regionalizing a shape parameter (see Table 3.3 for shape parameter for various distributions) rather than finding a regional flood distribution. Consequently, a subset of the methods normally used in this regional approach were employed.

The approach used involved: 1) identifying a homogenous region; 2) selecting a candidate distribution for this region; and 3) estimating the regional shape parameter for the region. Gages which comprise a homogenous region are considered not to have too large a spread in the L-moment version of the coefficient of variation within the region, termed L-CV. L-CV is computed as the ratio of the second L-moment to the sample mean.

A candidate distribution is selected from among the distributions shown in Table 3.3 by measuring how well the distribution explains the regional sample estimates of the L-kurtosis, the ratio of the fourth and second L-moments. This is done by comparing the kurtosis of the regional average distribution to the L-kurtosis averaged over all sites. A number of different distributions may provide reasonable fit. In this case, if the quantiles estimated by any of the different distributions are not significantly different, then any of the distributions may be selected. Otherwise some other criterion needs to be developed to select a distribution.

5.2.2 Estimating the regional shape using generalize least squares regression

The TAG recommended investigating the relationship between drainage area and distribution shape factor using generalized least squares regression (GLS). The regression relationship explored is simply written as:

$$\hat{\theta}_i = b_0 + b_1(DA_i) + e_i \tag{5.1}$$

where the θ_i are either the estimated skew or shape factor, at each station, DA_i is the drainage area (or the log of the drainage area) in square miles, e_i is the regression residual and b_0 and b_1 are regression parameters to be estimated. Both ordinary (OLS) and generalized (GLS) types of least squares analysis were used to estimate the coefficients. In OLS, the magnitude of the residual error is assumed to be independent of prediction and the residuals are uncorrelated. The GLS methodology can account for any estimated covariances (i.e., correlation of relationship between prediction and residual error) for the regression residuals. These covariances are modeled in the application to shape factors and skew coefficients as being the sum of **model (spatial)** and **time sampling (record length)** errors (Stedinger and Tasker, 1986). The time sampling errors are computed based on the at-station record length and the estimated inter-station covariances. The spatial sampling error is assumed to be only a function of the at-site estimate (no covariance between model errors) and is obtained by an iterative solution of the GLS “normal” equations (see Stedinger and Tasker, 1986).

5.3 Expected moments algorithm

The TAG recommended application of the expected moments algorithm (EMA, see Cohn et al., 1997) as an alternative to the regression approach described in section 3.3.4. The method is an extension of the standard moments approach described in section 3.3.1. The method estimates the sample moments of a full distribution by summing contributions from the uncensored data and the expected contribution from the truncated distribution as shown in Figure 5.1. For example consider the following modification to the standard moment estimate (equation 3.8) of the mean when a portion of the data has been censored:

$$\sum_{j=m+1}^{j=N} \frac{X_j}{(n-m)} + m(\overline{X}_c) = \int x f(x|\alpha,\beta,\kappa)dx \tag{5.2}$$

where n is the total number of observations, m is the number of values below the censoring threshold, α, β, κ are parameters of distribution defined by $f(x|\alpha,\beta,\kappa)$, and $\overline{X}_c$ is the expected or “average” value of the distribution below the censoring threshold. The left hand side of this equation is a substitute for the usual sample average, Σ(X/n), where the product m($\overline{X}_c$) is substituted for the m values below the threshold. The method requires an iterative solution because the distribution parameters □α, β□, κ needed to calculate $\overline{X}_c$ are not known. An iterative solution is performed where an initial guess is made of the parameters to calculate $\overline{X}_c$ and then left hand and right hand sides of the equation are compared for equality. Adjustments are made to the parameters until both sides of the equations agree. In the case of a three parameter distribution shown, two more moment equations in addition to equation (5.2) are provided, giving three equations and three unknowns to solve for in this iterative process.

EMA was applied in this study to only the log-Pearson III distribution. A threshold level was chosen such that data below the median values is censored as in the regression approach described in the section 3.2.4.

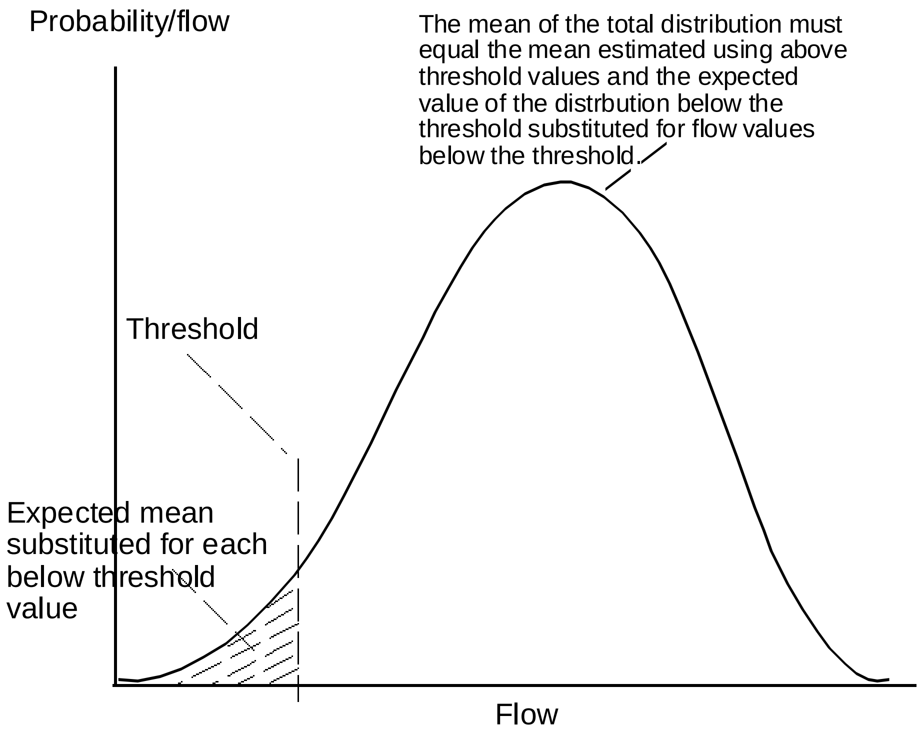

Figure 5.1: Expected moments algorithm

6. Sensitivity analysis results

6.1 Introduction

The TAG recommend a sensitivity analysis be performed comparing distributions obtained using at-site, regional shape and EMA estimation methods. This section provides a summary of sensitivity analysis results. The initial step in performing this analysis was to define a region for establishing regional shape parameters and skew coefficients as is described in section 6.2. Section 6.3 describes the results of the analysis performed to select best distributions within the region based on L-moment/regional flood procedures defined by Hosking and Wallis (1997). The investigation of the variation of regional shape parameters within the identified region using generalized least squares analysis is discussed in 6.4. Section 6.5 reports the results of estimating the log-Pearson III distribution from data censored below the median using EMA. The sensitivity of quantile estimates to distributions estimated using at-site, regional shape estimation, and EMA methods is summarized in section 6.6.

6.2 Region identification

Regions needed for obtaining regional shape parameters and skew were obtained using the sample L-moments of the peak annual 1-day stream flow at the study area gages. A homogenous region was identified by using sample L-moments to compute discordancy and heterogeneity statistics for aggregations of gages (Hosking and Wallis, 1997). Heterogeneity and discordancy statistic magnitudes for gage groupings indicate whether or not a particularly grouping defines a useful region for estimating flood quantiles.

In some respects, the resulting aggregation of gages does not correspond to the common concept of a region. Rather, a region is more of a mathematical concept where the flow record for the aggregation of gages have similarly shaped distributions but do not necessarily define a contiguous area on a map.

Typically a region corresponds to the situation where the sample L-moment coefficient of variation (L-CV) does not vary greatly among the aggregated gages as is shown for the study area in Figure 6.1. The gages which apparently deviate from the plot trend (Anoka, St. Paul and Mankato), also are identified as either discordant or not consistent with a homogenous region when aggregated with the remaining stations. Possibly, the deviation of these gages from the general trend is due to the influence of snowmelt floods. Further, investigation of these gages is warranted to determine if this is true. Consequently, as shown in Table 6.1, regions separating these stations were used in computing the regional average shape parameters.

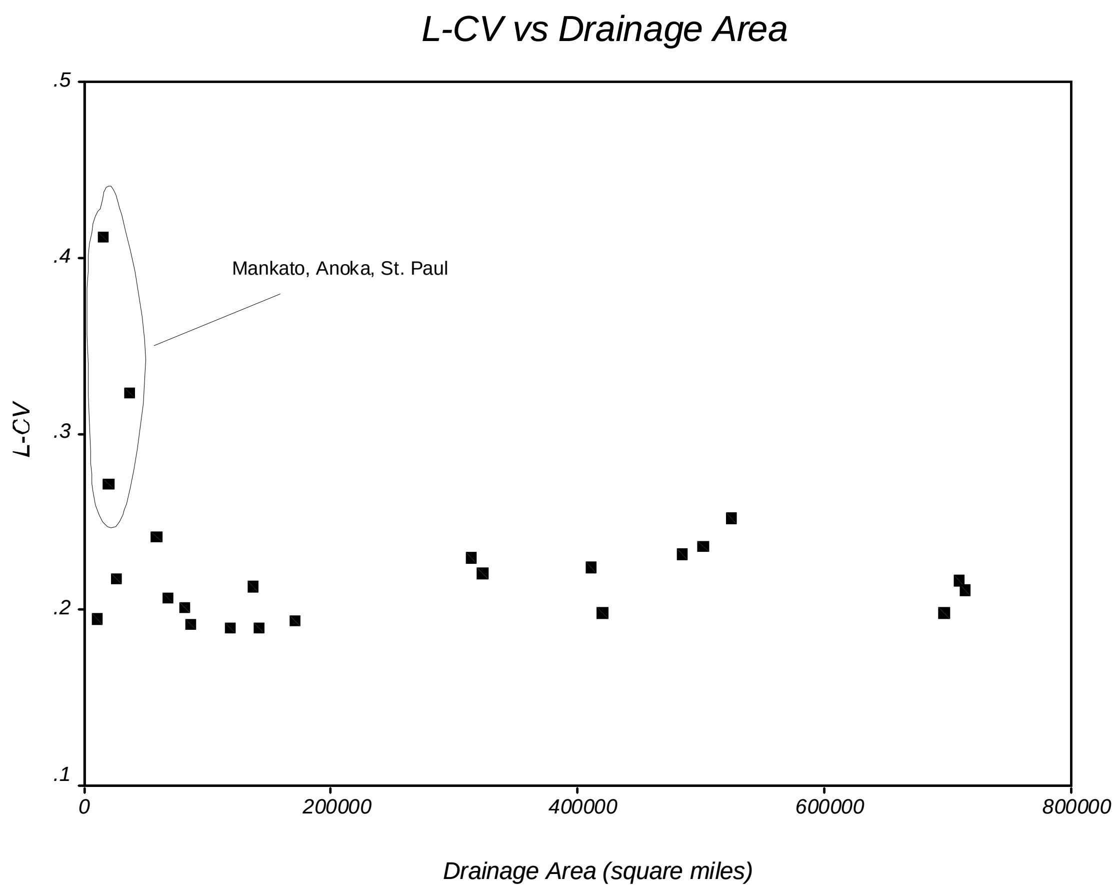

Figure 6.1: Drainage area versus L-moment coefficient of variation

Table 6.1: L-moments and regions for gages

Location	Mean (cfs)	[1]L-CV	[2]L-skew	[3]region
Sioux City	158040.	.2292	.0922	1
Omaha	156540.	.2204	.0865	1
Nebraska City	179010.	.2237	.0912	1
St Joseph	178220.	.1978	.1398	1
Kansas City	229460.	.2316	.2187	1
Booneville	280990.	.2359	.1925	1
Hermann	342870.	.2518	.2096	1
Anoka	32228.	.2717	.1377	2
St Paul	45101.	.3236	.2413	2
Mankato	20388.	.4120	.3252	2
Winona	94395.	.2412	.1782	1
McGregor	114960.	.2064	.1925	1
Muscoda	47027.	.1954	.0267	1
Dubuque	130403.	.2013	.0912	1
Clinton	141869.	.1913	.0911	1
Keokuk	188542.	.1900	.0876	1
Hannibal	211625.	.2130	.0984	1
Louisiana	246851.	.1899	.1381	1
Meredosia	66007.	.2177	.0664	1
Alton	290071.	.1939	.1349	1
St Louis	519299.	.1979	.1060	1
Chester	566141.	.2169	.0839	1

[1]L-moment coefficient of variation
[2]L-moment skew coefficient
[3]Region refers to a grouping of gages, not a geographic area

6.3 Candidate distribution selection, parameter estimates

A candidate distribution was selected from those indicated in Table 3.3 using the regional L-moment procedures. The generalized normal distribution was found acceptable based on the L-Kurtosis test goodness-of-fit statistic (see Hosking and Wallis, 1997). The test statistic for the GEV distribution also was very close to the acceptance criteria. The parameters for each of these distributions is shown in Tables 6.2 and 6.3.

6.4 Regional shape investigation results

6.4.1 Estimating time sampling error for GLS

The TAG recommended application of GLS regression to investigate the variation of shape or skew coefficient across the region identified in section 6.2. The application requires decomposing the residual error covariance (see section 5) into model and time sampling errors. The model error is determined based on an iterative solution of the GLS "normal" equations (see Stedinger and Tasker, 1986) once the time sampling error is identified.

The time sampling error in the GLS application would be the mean square error given for station skew in Bulletin 17B (see Table 1, pg. 14), except additional errors exist because of inter-station correlation. A somewhat more involved analysis is needed in a regional analysis because of inter-station correlation. In this case the time sampling error is computed as follows:

1. Estimate the distributions (log-Pearson III, generalized normal and GEV) using the methods previously described for each station;
2. Compute regional average skew or shape;
3. Transform the observed annual daily maximum flows to normal deviates based on the estimated distributions in (1);
4. Compute covariance matrix of transformed flows;
5. Obtain transformed flow traces of record length equal to the observed record length at each station given the joint normal distribution defined by steps 2 and 3 using Monte Carlo simulation;
6. Compute untransformed traces using the generated normal deviates, at-site location and scale parameters from step 1 and regional shape or skew parameters from step 2;
7. Estimate the distribution shape or skew coefficients at each gage site from the traces;
8. Repeat steps 4-7, retaining inter-station covariance (and mean square error) statistics of the shape or skew coefficient;
9. Repeat step 8 until the estimated inter-station covariance matrix stabilizes.

Application of the above steps was fairly straightforward except for decomposing the covariance matrix in step 4 when obtaining traces of normal deviates. A decomposition algorithm used by Lane (1989) in computer program LAST was employed to handle the inevitable problems in the decomposition of a sample covariance matrix which is not diagonally dominant.

The results were verified by noting that the inter-station covariance matrix of the generated flow traces, and the at-sites statistics and parameters of the marginal distributions obtained in step 1 were

Table 6.2: Parameters for generalized normal distribution (see Table 3.3)

Location	α	ξ	k
Sioux City	155116	62710	-.093024
Omaha	154204	59799	-.077994
Nebraska City	175923	69351	-.088830
St Joseph	172939	60477	-.173334
Kansas City	213744	87793	-.347327
Booneville	264179	111041	-.296182
Hermann	318118	143079	-.336288
Anoka	31092	14908	-.151505
St Paul	39693	23514	-.438226
Mankato	16253	12524	-.602217
Winona	88954	38367	-.278151
McGregor	110914	39963	-.200471
Muscoda	47969	15679	.119727
Dubuque	128232	45566	-.095072
Clinton	139659	47139	-.093525
Keokuk	185960	62257	-.082792
Hannibal	207147	78134	-.114233
Louisiana	242542	80265	-.107053
Meredosia	65986	24793	-.001670
Alton	284915	96363	-.106689
St Louis	508791	181261	-.154016
Chester	568054	215129	-.031767
Thebes	583060	212777	.011997

Table 6.3: Parameters for GEV distribution (see Table 3.3)

Location	□ α	ξ	k
Sioux City	133698	59893	.203
Omaha	133689	57498	.216
Nebraska City	152207	66360	.207
St Joseph	152787	55755	.137
Kansas City	186156	75292	.002
Booneville	228654	97210	.040
Hermann	272981	123246	.010
Anoka	26090	13875	.155
St Paul	32546	19472	-.064
Mankato	12686	9793	-.176
Winona	76603	33837	.054
McGregor	97713	36415	.115
Muscoda	42294	16514	.391
Dubuque	112678	43479	.202
Clinton	123562	45011	.203
Keokuk	164632	59733	.212
Hannibal	180631	73924	.186
Louisiana	215243	76182	.192
Meredosia	57292	24679	.282
Alton	252138	91476	.192
St Louis	448025	168519	.153
Chester	493257	211217	.256
Thebes	508175	213135	.294

preserved in the simulation. Furthermore, the mean square error estimates of the skew coefficients obtained agreed with those published in Bulletin 17B (Table 1, pg. 14). The at-site inter-station correlation values used in the simulation, time sampling error covariance matrix for the skew coefficient, and the generalized normal and GEV shape parameters are given in Appendix D.

6.4.2 Regression estimates

The results obtained from the application of GLS were verified by comparison with estimates obtained using weighted least squares (WLS) and ordinary least squares regression (OLS). The difference between OLS, WLS, and GLS is in the estimate of the residual error covariance matrix. GLS reduces to OLS when the residual errors are not a function of the prediction, or equivalently, the site considered. WLS is obtained when there is no inter-station correlation between peak annual flows, but the residual error magnitude is a function of the prediction. The initial testing of the software developed to perform the GLS analysis used these differences by comparing GLS: 1) to OLS; and, 2) with a weighted least square (WLS) regression program (RSKEW, Tasker, 1986) using the same assumptions for the residual errors. The equivalence obtained between OLS and GLS for some standard problems provided a simple verification of the GLS solution algorithm.

The comparison with WLS was performed by using an equivalent residual error model in the GLS data. In this case, the GLS covariance errors are set to zero, and only the residual error variances are allowed to change with prediction. Table 6.4 provides a comparison of the results obtained with the GLS software, WLS (RSKEW software) and OLS for an example data set provided with the RSKEW software. Examination of the results reveals that the GLS software provides very close answers with that obtained from the WLS software. The OLS software results shown differ because the residual error model assumed is different than assumed in WLS or GLS.

The application of the GLS software to the data in Table 6.5 was disappointing as shown in Table 6.6. Application to the skew coefficient resulted in no admissible value for the model error variance ((^{2}). Tasker and Stedinger (1986) have noted that this is possible. Effectively, skew is not related to drainage area and might be taken as a constant over the region. The application to regional shape parameters was more successful in that a model error could be estimated. However, the R^2 attributable to the regressions is not impressive.

Table 6.4: Comparison of results, U.S. Geological Survey WLS versus GLS software (data from Tasker, 1986)

Regression	b_0 (1)	b_1 (1)	b_2 (1)	b_3 (1)	(^{2} (2)	(2) (3)
GLS[1]	-0.6113	0.4788	0.1038	0.1451	0.088	0.026
WLS[2]	-0.6192	0.4693	0.1058	0.1453	0.075	0.026
OLS[3]	-0.5445	0.5805	0.0848	0.1539	-------	------

(1) regression: $2^* = b_0 + b_1x_1 + b_2x_2 + b_3x_3$
(2) model sampling error variance
(3) generalized least square average regression prediction error variance (regression line error)
[1]generalized least squares software
[2]weighted least squares software (Tasker, 1986, Tasker and Stedinger, 1986)
[3]ordinary least squares

Table 6.5: Stations, drainage area and shape parameters used in least squares analysis (period of record 1933-1996)

Station	Name	Drainage Area (square miles)	skew (1)	gnormal (2)	gev (3)
1	Sioux City	314580	-0.13	-0.25	0.08
2	Omaha	322800	-0.13	-0.20	0.12
3	Nebraska City	410000	-0.19	-0.19	0.13
4	St. Joseph	420300	-0.10	-0.19	0.13
5	Kansas City	485200	0.10	-0.35	0.00
6	Booneville	501200	-0.02	-0.29	0.04
7	Hermann	524200	0.04	-0.34	0.01
8	Anoka	19600	-0.25	-0.16	0.15
9	St. Paul	36800	0.20	-0.46	-0.08
10	Mankato	14900	-0.07	-0.65	-0.21
11	Winona	59200	0.09	-0.30	0.04
12	Dubuque	82000	-0.18	-0.07	0.22
13	Clinton	85600	-0.35	-0.04	0.25
14	Keokuk	119000	-0.31	-0.04	0.25
15	Hannibal	137000	-0.19	-0.01	0.27
16	Louisiana	141000	-0.11	-0.10	0.19
17	Meredosia	26030	-0.50	0.06	0.34
18	Alton	171300	0.12	-0.10	0.20
19	St. Louis	697000	-0.18	-0.05	0.24
20	Chester	708600	-0.30	-0.03	0.26

(1) log-Pearson III skew coefficient
(2) Generalized Normal shape parameter
(3) Generalized Extreme Value shape parameter

Table 6.6: Least square regression results relating drainage area to skew and shape factors

Method	b_0 (1)	b_1 (1)	(2)	s_2^2 (3)	s_e^2 (4)	(^{2} (5)	(2) (6)	R^2 (7)
lpiii ols[1]	-0.3577	0.0453	-0.12	0.0311	0.0305	---------	---------	0.02
lpiii gls[2]	-0.3612	0.0456	-0.12	0.0311	0.0305	0.0000	.0094	0.02
gnormal ols[3]	-0.5123	0.0625	-0.18	0.0301	0.0292	---------	---------	0.03
gnormal gls[4]	-0.5120	0.0624	-0.18	0.0301	0.0289	0.0215	0.0031	0.04
gev ols[5]	-0.0834	0.0413	0.13	0.0182	0.0177	---------	---------	0.03
gev gls[6]	-0.0899	0.0426	0.13	0.0182	0.0177	0.0085	0.0019	0.03

[1]log-Pearson III skew, ordinary least squares regression
[2]log-Pearson III skew, generalize least square regression
[3]generalized normal shape factor, ordinary least squares regression
[4]generalized normal shape factor, generalized least squares regression
[5]generalized extreme value shape factor, ordinary least squares regression
[6]generalized extreme value shape factor, generalized least squares regression
(1) regression: $2^* = b_0 + b_1(\log_{10}(DA))$, 2^* is estimated skew or shape factor, DA is drainage area in square miles
(2) mean of skew or shape factors (see Table C.1)
(3) variance of skew or shape factors (see Table C.1)
(4) regression standard error squared, $\Gamma(-2^*)^2/(N-2)$, where N is number of observations of , in Table 3.3
(5) generalized least square model error variance
(6) generalized least square average regression prediction error variance (regression line error)
(7) $R^2 = 1-(s_e^2/s_2^2)$

6.4.3 Regional shape and skew coefficient estimates

The GLS application produced no useful relationship between shape or regional skew coefficient and drainage area. Consequently, the TAG recommendation was to employ a constant regional value in applications of regional shape estimation.

The approach taken was to estimate the average regional shape or skew coefficient over two different regions, the region identified as relatively homogenous and the remaining Minnesota stations (see figure 6.1) as is shown in Table 6.7. Included in these results are the regional skew coefficients obtained from an application of EMA (see section 6.5).

The sensitivity analysis involves comparing estimates obtained using regional shape estimation and Bulletin 17B. Regional skew values needed for the Bulletin 17B applications are shown in Table A.1. Gages used in estimating averages were selected based on both the discordance of the Minnesota stations and by considering the variation of skew on the map provided in Bulletin 17B. Calculation of the regional skew mean square error is more difficult to justify. The mean square error was assumed to be the average of the squared errors from the mean skew for the region identified as homogenous from the regional L-moment application (see section 6.2, Table 6.8). Computing regional and average skews in this manner corresponds to one of the recommended techniques in Bulletin 17B. However, the inter-station proximity does not correspond to the guideline recommendations of at least 25 gages, or all gages within a 100-mile radius.

Table 6.7: Regional skew and shape parameters substituted for at-site estimates in method comparisons (full period of record)

Station Locations	[2]Bulletin 17B	[3]EMA	[4]GNORMAL	[5]GEV
[1]Minnesota	-0.340	0.360	-0.397	-0.028
All other stations	-0.260	0.360	-0.127	0.178

[1]Anoka, St. Paul and Mankato
[2]Regional skew substituted for adopted (or weighted skew) in Bulletin 17B procedure
[3]Regional skew obtained from expected moments algorithm, and substituted for adopted (or weighted skew) in Bulletin 17B procedure
[4]Generalized Normal distribution regional shape factor substituted for at-site estimate
[5]Generalized Extreme Value distribution regional shape factor substituted for at-site estimate

Table 6.8: Regional skew coefficients for standard application of Bulletin 17B (full period of record)

Station Locations	Regional Skew	Mean Square Error
Minnesota[1]	-0.34	0.05
Upper Mississippi[2]	-0.26	0.05
Missouri	-0.22	0.05

[1]Anoka, St. Paul and Mankato
[2]Mississippi River Basins minus Anoka, St. Paul and Mankato

6.5 Application of expected moments algorithm

The application of the expected moments algorithm (EMA) to flow records censored below the median resulted in skew values that are considerably larger than was obtained using the Bulletin 17B method (see Table 6.9). This difference results because of the flexibility available in fitting a three parameter distribution to the upper tail of the plotted points. For example, figure 6.2 shows how the difference between using the full data and the upper half of the data influences the resulting tail of the LPIII distribution. Consequently, application of a three parameter distribution to the top half of the data will result in greater variation in the estimate of extreme quantiles (e.g., the 1% chance flow) than either application of a two parameter distribution/regression estimation pairing (see section 3.2) or the Bulletin 17B methodology.

Table 6.9: Sample statistics for LPIII distribution estimated from data censored below median using EMA

Location	*mean	*standard deviation	*skew
Sioux City	5.2017	.1298	0.8457
Omaha	5.2132	.1100	1.3176
Nebraska City	5.2685	.1181	0.7765
St Joseph	5.2655	.1047	1.4767
Kansas City	5.3617	.1316	1.4050
Booneville	5.4166	.1723	0.2944
Hermann	5.4877	.2010	-.0325
Anoka	4.5067	.1556	0.6412
St Paul	4.6213	.2067	0.5217
Mankato	4.2261	.2860	0.3420
Winona	4.9483	.1723	0.1558
McGregor	5.0506	.1329	0.7764
Muscoda	4.6483	.1541	-.4776
Dubuque	5.1119	.1275	0.2748
Clinton	5.1618	.1092	0.4653
Keokuk	5.2642	.1316	0.0789
Hannibal	5.3355	.1200	0.4857
Louisiana	5.3684	.1479	-.0907
Meredosia	4.8231	.1301	0.1567
Alton	5.4229	.1712	-.3756
St Louis	5.6905	.1542	-.2319
Chester	5.7329	.1621	-.3562

*Statistics of log-flow values

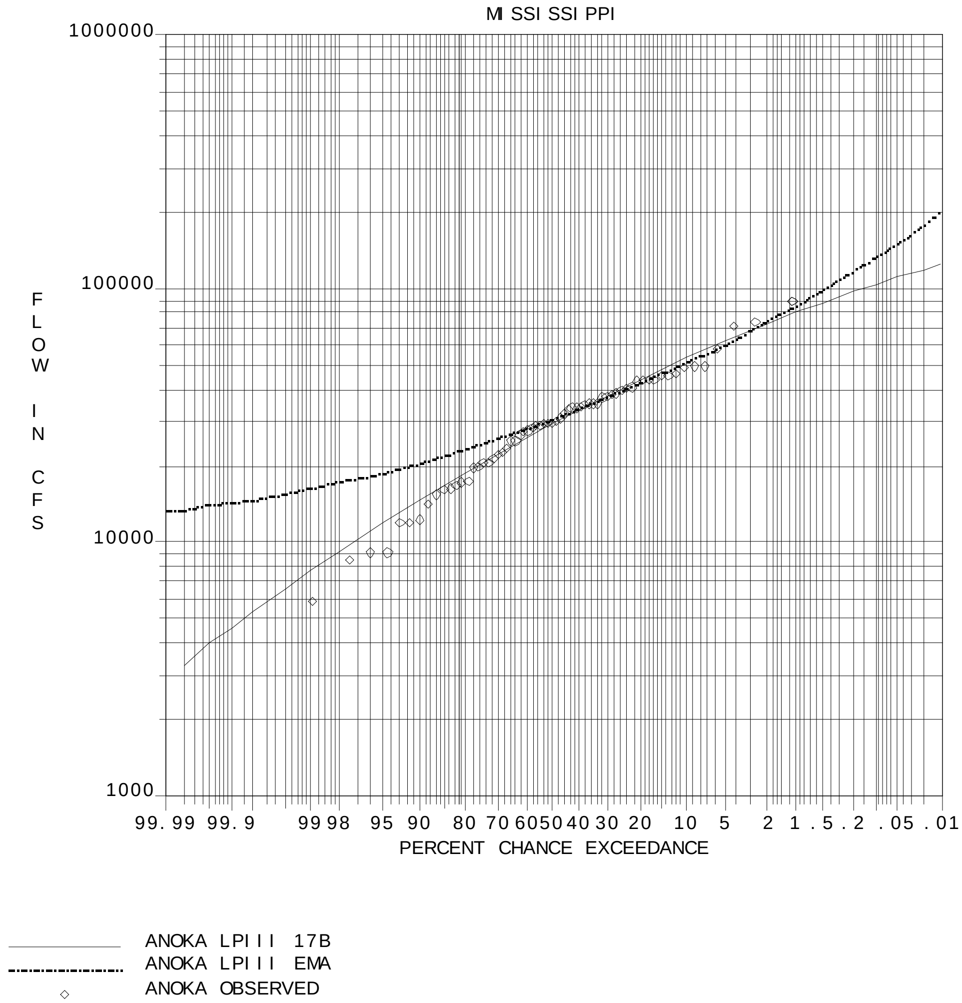

Figure 6.2: Comparison of Bulletin 17B and fit to data censored below median using EMA at Anoka, Mississippi River

6.6 Distribution/estimation method pairing comparisons

The following flood distribution/estimation pairings were investigated:

(1) Bulletin 17B, log-Pearson III distribution/standard moment estimation either with at-site estimate of skew or the weighted (adopted) skew value (lpiii, 17B);
(2) Regional shape estimation applied to Bulletin 17B; in this case regional skew is substituted for the weighted (adopted) skew value (lpiii, 17B, regional skew substituted for weighted skew);
(3) log-Pearson III estimation with EMA applied to data censored below median value (lpiii, with expected moments);
(4) log-Pearson III estimation with EMA applied to data censored below median value, regional shape estimation where regional skew is substituted for at-site skew value (lpiii, with expected moments, regional skew substituted for weighted skew);
(5) generalized normal distribution/L-moment estimation (generalized normal, L-moments)
(6) generalized extreme value distribution/L-moment estimation (gev, L-moments)
(7) generalized normal distribution/L-moment estimation, regional shape estimation where regional generalized normal shape is substituted for at-site value (generalized normal, L-moments, regional shape substituted for at-site estimate)
(8)generalized extreme value distribution/L-moment estimation, regional shape estimation where regional generalized extreme value shape is substituted for at-site value (gev, L-moment, regional shape substituted for at-site estimate)

The 17B guidelines were applied to method (1) with different assumptions regarding regional skew. The first application assumed regional skew is not available. The second application assumed a regional skew can be estimated and used to compute the adopted skew. The first application with out regional skew is not unreasonable given the long-record lengths available in the study. The mean square error of the station skew is small given these record lengths in comparison with that of a region skew obtained from the map available in the guidelines. The second application to method (1) using a regional skew was practical, at least for region 1 shown in Table 6.1, given the reasonable number of stations available for calculating mean square error.

The results of the comparisons with the estimates obtained using Bulletin 17B, no regional skew, at individual gages are shown in Tables 6.10-6.12 for the (1/25), (1/100) and (1/500) chance exceedance events. See Appendix C for plots providing comparisons of the different estimates of flood distributions with plotting positions for the observation at selected gages.

The average difference over all the gages is shown in Table 6.13 for a range of exceedance probabilities. As might be expected, the differences increase with decreasing exceedance probability. As can be seen from Table 6.13, a maximum difference averaged over all gages was obtained between Bulletin 17B and the GEV distribution of about 13% at (1/100) and 18% at (1/500) chance exceedance probabilities. Comparisons made with flood distribution obtained with Bulletin 17B using regional skew values resulted in about the same differences on the average as is shown in Table 6.14.

Table 6.10: Method Comparison with LP III no regional skew, (1/25) chance exceedance probability flow

station	flow 17B	2	3	4	5	6	7	8
Sioux City	290855.	4.54	-.41	-4.39	-2.61	-2.43	-5.10	-6.38
Omaha	282277.	3.44	-1.19	-7.10	-2.56	-2.40	-4.53	-5.58
Nebraska City	323460.	3.81	-1.53	-4.66	-1.90	-1.73	-4.30	-5.54
St Joseph	311846.	-3.97	-.97	-7.29	-1.64	-1.43	-7.74	-10.42
Kansas City	439382.	-3.57	-.23	-7.81	.09	.13	-13.88	-18.88
Booneville	534734.	-2.45	1.58	2.41	.33	.48	-11.38	-15.86
Hermann	686222.	-4.39	.20	6.34	-.69	-.61	-14.30	-19.28
Anoka	65031.	2.09	-.68	-3.56	-1.47	-1.22	4.58	.77
St Paul	105195.	.00	-1.05	-3.30	-.99	-1.11	-7.04	-14.93
Mankato	55259.	4.29	3.84	4.22	3.22	1.89	-15.58	-26.17
Winona	182404.	-2.36	-.49	2.12	-.85	-.65	-11.22	-15.35
McGregor	205015.	-2.77	.66	-2.93	.68	.83	-10.24	-14.43
Muscoda	76452.	6.72	1.81	12.92	.02	-.02	2.65	4.00
Dubuque	221486.	-.38	.30	1.09	-.19	-.03	-2.49	-3.67
Clinton	236483.	-.25	-1.04	-1.83	-.38	-.22	-2.59	-3.74
Keokuk	311039.	1.68	1.22	4.04	.23	.38	-1.72	-2.74
Hannibal	368168.	1.43	-.43	-1.46	.11	.30	-2.87	-4.35
Louisiana	416577.	-1.82	.69	5.92	.20	.41	-5.72	-8.35
Meredosia	116110.	2.40	-1.64	.29	-1.61	-1.50	-1.96	-2.23
Alton	494784.	-4.06	1.16	11.76	-.16	.06	-5.88	-8.43
St Louis	883690.	-1.13	.35	7.71	-.40	-.21	-3.84	-5.51
Chester	995218.	1.30	-.47	9.09	-.86	-.70	-2.63	-3.59
Thebes	991624.	1.84	-.32	6.39	-.64	-.52	-1.25	-1.65

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

Table 6.11: Method Comparison with LPIII no regional skew, (1/100) chance exceedance probability flow

station	flow 17B	2	3	4	5	6	7	8
Sioux City	345308.	9.14	10.31	-.17	-2.45	-3.34	-6.40	-9.37
Omaha	334766.	6.94	10.36	-6.02	-2.93	-3.90	-6.05	-8.74
Nebraska City	382761.	7.67	5.95	-2.03	-1.69	-2.60	-5.49	-8.41
St Joseph	384611.	-8.13	6.58	-10.60	-4.00	-4.01	-13.49	-17.69
Kansas City	558899.	-7.26	10.98	-9.88	1.17	2.61	-20.76	-27.28
Booneville	672945.	-4.96	6.23	8.23	.92	1.91	-17.39	-23.49
Hermann	892968.	-8.93	-.07	13.98	-1.56	-.26	-22.32	-28.75
Anoka	80970.	4.22	7.51	.21	-1.84	-1.90	8.02	1.28
St Paul	141244.	.00	7.02	1.40	.19	2.20	-9.39	-19.99
Mankato	77667.	8.75	18.05	19.05	10.21	13.70	-19.49	-32.47
Winona	230582.	-4.78	1.35	7.45	-1.38	-.67	-17.43	-23.16
McGregor	252286.	-5.64	7.34	-1.71	2.06	3.01	-15.42	-21.25
Muscoda	85674.	13.35	4.55	30.05	1.47	-.45	5.76	5.76
Dubuque	262065.	-.77	3.63	5.52	-.89	-1.76	-4.53	-7.33
Clinton	278514.	-.50	1.75	-.13	-1.07	-1.91	-4.60	-7.31
Keokuk	360149.	3.38	5.05	11.73	1.01	.09	-2.14	-4.78
Hannibal	435334.	2.89	4.16	1.65	.35	-.43	-4.39	-7.58
Louisiana	498929.	-3.70	1.07	12.97	.22	.17	-9.27	-13.53
Meredosia	136923.	4.84	.95	5.48	-2.54	-3.86	-3.10	-4.95
Alton	604985.	-8.33	-1.88	21.35	-2.16	-2.26	-11.12	-15.23
St Louis	1055112.	-2.30	-.10	16.44	-1.39	-1.99	-6.83	-10.11
Chester	1180849.	2.63	-1.15	20.24	-1.55	-2.58	-4.36	-6.96
Thebes	1164117.	3.71	.28	15.50	-1.23	-2.48	-2.20	-4.19

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

Table 6.12: Method Comparison with LPIII no regional skew, (1/500) chance exceedance probability flow

station	flow 17B	2	3	4	5	6	7	8
Sioux City	399034.	14.75	28.05	7.45	-1.27	-4.41	-6.83	-12.60
Omaha	387390.	11.16	30.44	-2.30	-2.47	-5.74	-6.85	-12.27
Nebraska City	441689.	12.35	18.75	3.54	-.58	-3.75	-5.94	-11.66
St Joseph	471474.	-12.93	19.61	-13.04	-6.74	-7.65	-19.59	-25.90
Kansas City	703275.	-11.52	30.17	-10.65	2.49	6.01	-27.58	-35.95
Booneville	835244.	-7.89	12.83	16.49	1.80	3.69	-23.32	-31.51
Hermann	1150501.	-14.12	-.57	24.02	-2.58	.29	-30.26	-38.39
Anoka	98226.	6.79	21.26	7.30	-1.45	-2.64	12.78	1.94
St Paul	184511.	.00	20.61	9.87	2.42	7.86	-11.02	-24.73
Mankato	104568.	14.24	40.95	43.00	20.32	34.21	-22.00	-37.59
Winona	287277.	-7.61	4.33	15.21	-1.75	-.73	-23.53	-31.34
McGregor	307222.	-8.99	17.75	.92	3.75	5.59	-20.70	-28.67
Muscoda	93670.	21.41	7.81	54.11	3.86	-1.32	10.07	7.17
Dubuque	305590.	-1.23	8.71	12.11	-1.53	-4.53	-6.61	-12.02
Clinton	323283.	-.80	6.75	3.38	-1.60	-4.54	-6.54	-11.83
Keokuk	409517.	5.42	10.45	22.58	2.31	-.86	-2.19	-7.56
Hannibal	504822.	4.63	11.86	7.25	1.06	-1.81	-5.60	-11.54
Louisiana	590907.	-5.91	1.49	22.31	.34	-.70	-12.95	-19.59
Meredosia	158015.	7.77	5.57	13.74	-2.97	-6.91	-3.75	-8.37
Alton	736154.	-13.25	-6.30	32.98	-4.69	-5.82	-16.88	-23.18
St Louis	1243358.	-3.67	-.74	28.05	-2.35	-4.74	-9.91	-15.69

Chester	1374086 .	4.21	-1.77	35.69	-1.89	-5.25	-5.79	-11.12
Thebes	1339766 .	5.95	1.49	28.38	-1.42	-5.26	-2.78	-7.54

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

Table 6.13: Method Comparison with LPIII no regional skew, average difference over all stations in predicted flow

exceedance probability	2	3	4	5	6	7	8
.5000	-.06	1.86	1.59	.47	.47	.47	.22
.2000	.05	-1.87	-1.70	-.03	-.10	-1.48	-2.83
.1000	.14	-1.71	-1.10	-.34	-.33	-3.37	-5.27
.0500	.24	-.49	.60	-.50	-.43	-5.10	-7.62
.0400	.28	.06	1.30	-.53	-.45	-5.61	-8.36
.0200	.40	2.16	3.90	-.51	-.47	-7.08	-10.63
.0100	.53	4.78	6.99	-.40	-.46	-8.37	-12.85
.0020	.90	12.59	15.76	.22	-.39	-10.77	-17.82

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

Table 6.14: Method Comparison with LPIII, regional skew, average difference over all stations in predicted flow

probability	2	3	4	5	6	7	8
.5000	.03	1.97	1.69	.57	.57	.57	.35
.2000	.02	-1.91	-1.73	-.07	-.14	-1.52	-2.61
.1000	-.01	-1.85	-1.24	-.48	-.47	-3.51	-4.97
.0500	-.03	-.72	.36	-.73	-.66	-5.35	-7.25
.0400	-.04	-.20	1.03	-.78	-.70	-5.91	-7.96
.0200	-.06	1.81	3.52	-.84	-.79	-7.49	-10.16
.0100	-.08	4.33	6.50	-.79	-.84	-8.90	-12.30
.0020	-.11	11.89	14.97	-.30	-.82	-11.61	-17.10

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

An additional comparison of interest is between the predicted stages at particular exceedance probabilities obtained with the different flood distribution methods. Rating curves for each of the gage locations were obtained from the Corps Districts involved in the study. Unfortunately, the rating curves do not cover a range that can be usefully applied to floods exceeding the (1/50) chance exceedance. Consequently, Table 6.15 displays the maximum difference of the estimates obtained with Bulletin 17B and the GEV distribution which exceeds 2.0 feet at the (1/25) exceedance probability at certain locations. More typically, differences are on the order of 1.0 feet or less.

The TAG recognized that the comparisons between methods resulted in differences of less than 10% at the 1% chance flow value, except for GEV estimates obtained with regional information. Glaring differences between methods exists only between some selected gages when considering the GEV-regional shape estimation. Consequently, the TAG did not feel the differences between the various methods was great enough to recommend deviating from the Bulletin 17B guidelines.

Table 6.15: Method Comparison with LP III no regional skew, (1/25) chance exceedance probability stage

station	stage 17B	2	3	4	5	6	7	8
Omaha	41.165	.30	.11	.67	.23	.22	.42	.53
Nebraska City	-1.000	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
St Joseph	814.006	.29	.07	.54	.12	.11	.57	.77
Kansas City	749.594	.30	.02	.67	.01	.01	1.18	1.61
Booneville	-1.000	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Hermann	-1.000	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Anoka	19.829	.16	.05	.27	.11	.09	.34	.06
St Paul	705.436	.00	.17	.53	.16	.18	1.13	2.37
McGregor	-1.000	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Dubuque	22.658	.05	.04	.14	.03	.00	.32	.48
Clinton	17.838	.03	.14	.24	.05	.03	.34	.49
Meredosia	-1.000	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
Alton	37.785	.44	.13	.37	.02	.01	.64	.92
St Louis	45.991	.28	.13	1.75	.10	.05	.94	1.18
Chester	49.994	.01	.00	-1.00	.00	.00	.01	.02
Thebes	45.759	-1.00	.05	-1.00	.11	.09	.21	.27

(2) lpiii, 17B, regional skew substituted for weighted skew
(3) lpiii, with expected moments
(4) lpiii, with expected moments, regional skew substituted for weighted skew
(5) generalized normal, L-moments
(6) gev, L-moments
(7)generalized normal, L-moments, regional shape substituted for at-site estimate
(8)gev, L-moment, regional shape substituted for at-site estimate

7. Regional Consistency

7.1 Introduction

The flood-quantile estimates obtained by the application of the recommended distribution/estimation pairing should have a regular variation along the study area river reaches to produce consistent flood profile estimates. The combined regional and at-sites estimates obtained by the method recommended in section 6 are not constrained by any regularity condition. Statistical sampling error alone could cause inconsistent flood quantiles and corresponding flood profile inconsistencies. Ideally, some simple smoothing procedure should be instituted where flood distribution parameters or moments are required to vary regularly with drainage area. As is discussed in section 7.2, finding a simple algorithm to obtain this regular variation is complicated by the influence of tributary flows which can cause apparent discontinuities in the variation of distribution statistics. Section 7.3 discusses a procedure for smoothing statistics recommended by the interagency and technical advisory groups (IAG and TAG) given the variation caused by statistical sampling error and the impact of confluences .

7.2 Observed variation of skew coefficient and shape parameter

The need to obtain regionally consistent distribution parameters and corresponding flood frequency estimates is apparent from examining the variation of at-site skew estimate on the main stem study area rivers shown in figure 7.1. As can be seen, there is significant variation in skew between stations. For example, the skew coefficient at Louisiana is -0.11 and 0.10 at Alton. Is this variation due to statistical sampling error or is there some influence due to the confluence of the Illinois River near Alton that explains the difference? If an algorithm is applied to obtain a regular variation of the skew coefficient, then the potential impact of a major confluence may be incorrectly discounted.

Equal periods of record were examined per TAG recommendations to ascertain if this observed variation in skew was due to unequal record lengths. Consideration of equal periods of record was thought to remove to some extent the apparent variation of the skew coefficient or some shape parameter due to sampling error; or at, least constrain each estimate to having the same sampling error. Consequently, the skew coefficient and the shape parameter for the generalized normal (see Table 3.3) distribution were plotted for equal periods of record as shown in figures 7.2 and 7.3. Inspection of the figures reveals no apparent reduction in the scattered distribution of skew coefficient values for the equal period of record analyzed. The variation of generalized normal shape parameters is also significant.

The skew apparently increases at major confluences irrespective of the period of record chosen as can be seen by a comparison of figures 7.1 and 7.2 (see confluence of: (1) the Minnesota and Mississippi at St. Paul; (2) the Illinois an Mississippi at Alton; (3) the Kansas and Missouri at Kansas City; (4) the Gasconade and Missouri at Hermann; and (5) the Missouri and Mississippi at St. Paul). However, the generalized normal distribution shape factor does not reveal the same impact of confluences. For example, compare shape factors at St. Paul and Alton to those estimated at Kansas City and Hermann. Consequently, simple plots of the skew coefficient or shape parameters along the river reaches does not provide consistent or conclusive evidence concerning the impact of confluences on the skew coefficient or shape parameter estimates.

7.3 Recommendations

The IAG and TAG recommended investigating the importance of confluences using the available unsteady flow model and using a simple method for obtain a regular variation of the log-Pearson III distribution depending on the results of this analysis. The unsteady flow period of record simulations will provide data on the coincidence of peak flows at major tributaries. A study of this coincidence may reveal some relationship between skew coefficient and tributary flows. If the tributaries are significant, then reaches of the river can be formed to set boundaries for smoothing distribution moments.

Values of the mean and standard deviation of the log flows will interpolated linearly with drainage area between gage sites in reaches identified by the analysis of coincidence. If no boundaries are identified, then the interpolation will be performed along entire study area reaches (the Upper Mississippi, Missouri and Illinois Rivers). If only one gage exists within a reach then the results of the period of record unsteady flow analysis may be used to estimate within reach variation. The skew coefficient will be taken as the weighted regional and at-site estimates obtained as recommend in section 6.4.3. If the analysis of coincidence reveal a significant impact of tributaries, then an average of the weighted skew values should be used within identified reaches.

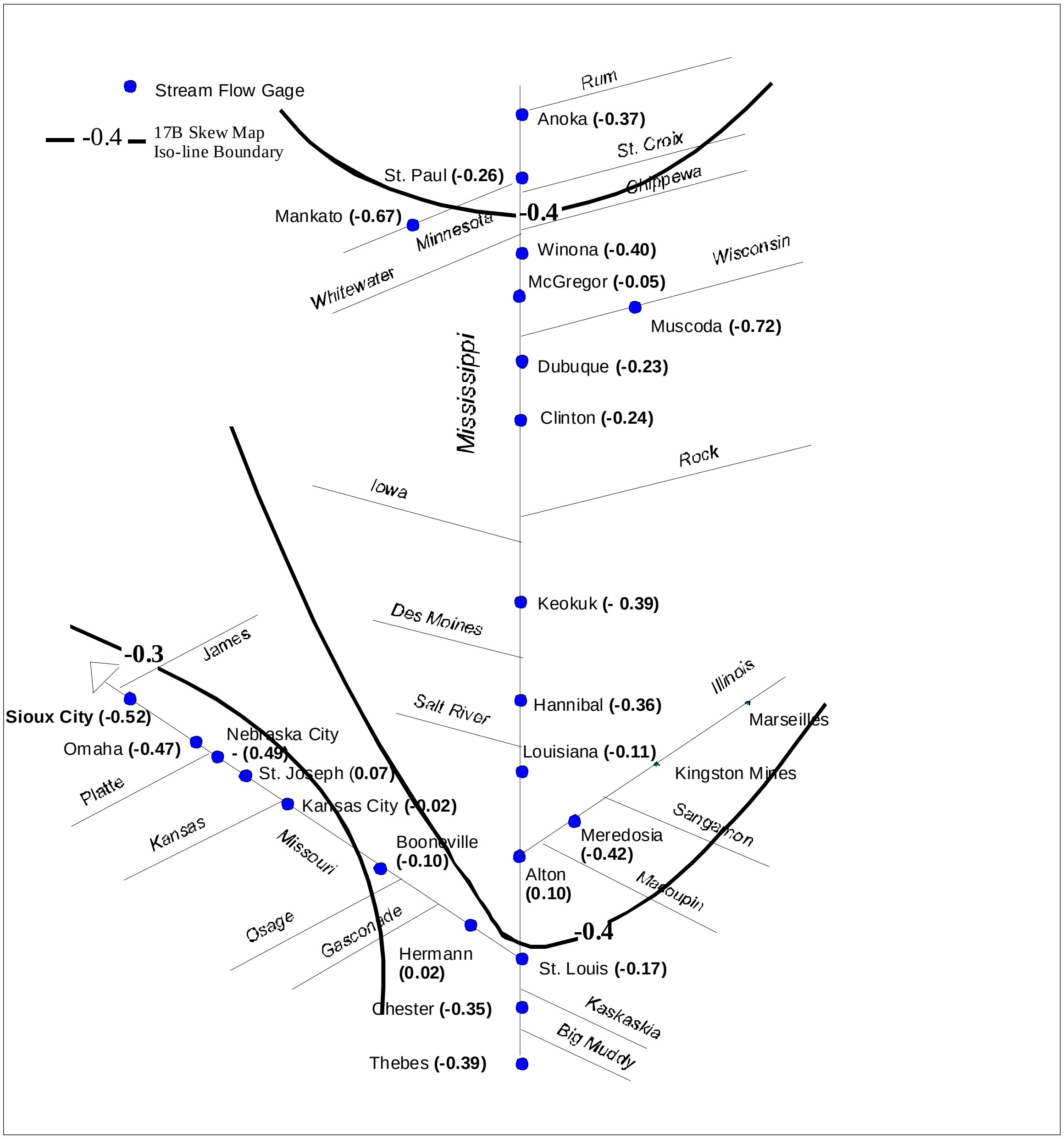

Figure 7.1: Variation of at-site skew coefficient full period of record, contours of skew from map in Bulletin 17B

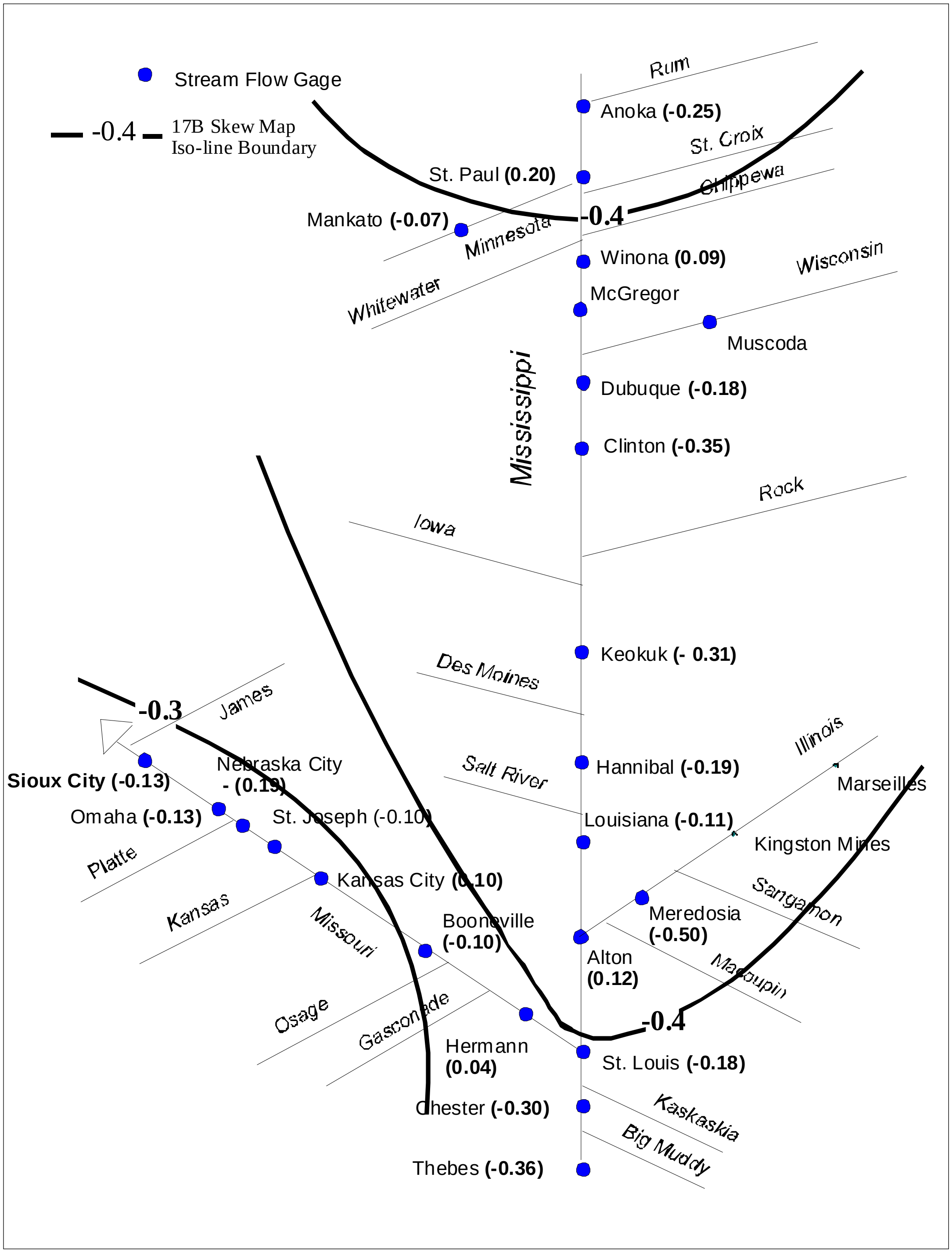

Figure 7.2: Variation of at-site skew coefficient, 1933-1996 period, contours of skew from map in Bulletin 17B

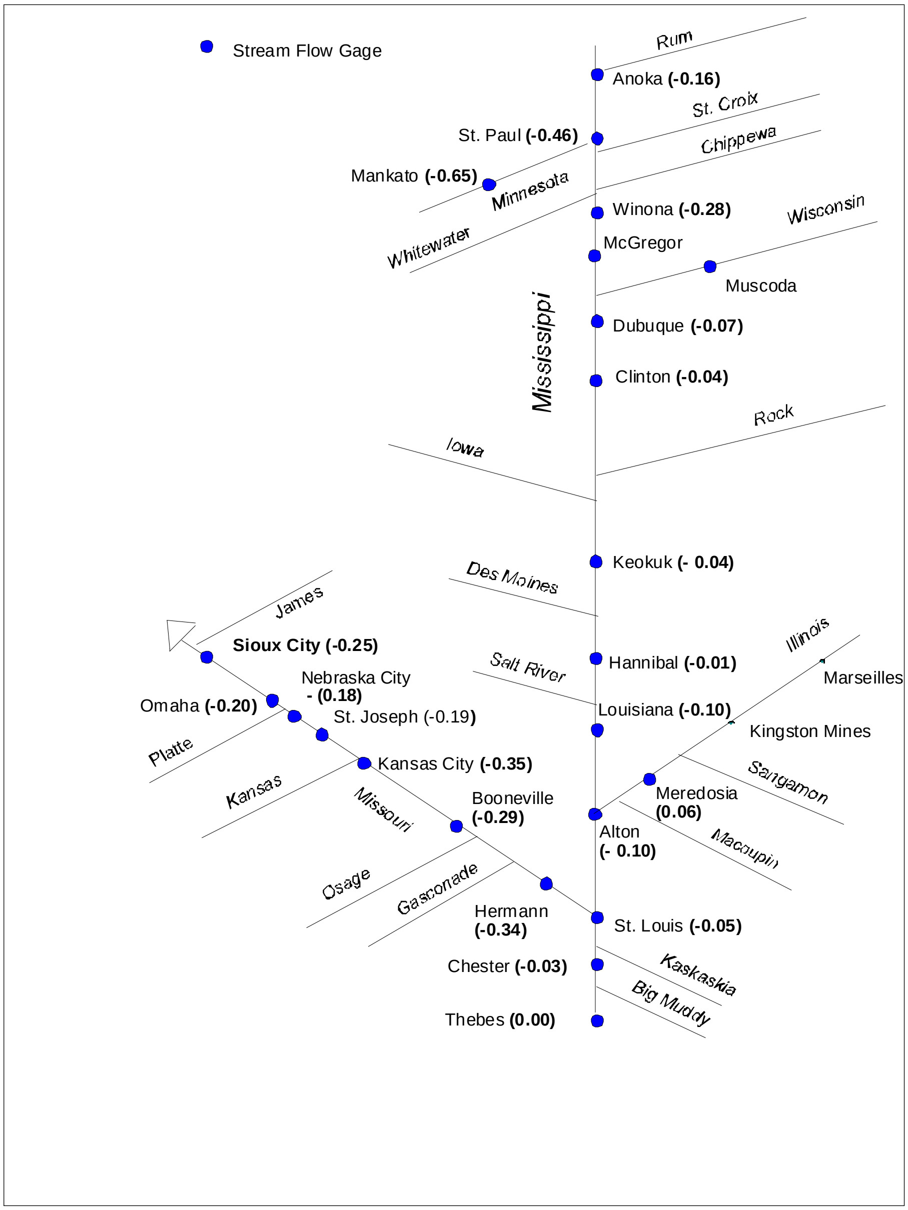

Figure 7.3: Variation of generalized normal shape parameter, 1933-1996 period

8. Analysis of non-randomness

8.1 Introduction

Flood quantiles computed from the latter part of flood record are greater than those obtained from the earlier portion of the record as was demonstrated with the forecast test used in the distribution selection investigation (see Section 4.2.2). The purpose of this section is to determine if this difference is due to random chance or is indicative of some non-randomness in the flood record.

The non-randomness may be due to either factors related to land use change or a small time-scale climatic cycle. The influence of land use change on flood frequencies is well known. For example, a number of studies have been performed in the study area to analyze the impact of farming on flood-frequencies on flow-frequency curves. However, extrapolating the conclusions of these studies which focused on very small drainage areas to the main stem Mississippi would not be appropriate.

Climatic variability certainly is a factor in the flood record over a geologic time scale. However, this variability has not been identified over time scales of engineering design interest. The gage flood record, on the order of a hundred years, is usually assumed to be approximately stationary over the design period. Still, there has been at least some discussion recently calling into question this assumption because of the influence of such factor as sea surface temperatures on climatic cycles.

Quantifying the impact of either land use change or climatic variability on the flood record, if these influences exist, is beyond the scope of this investigation. However, statistical methods exist that will provide some measure of the non-randomness of an observed set of data, such as a flood record. The non-randomness can be characterized by a trend or an episodic change, or a combination of both. The trend may be due to either the gradual change in land use or climate that has occurred over the period of record. An episodic change might be caused by a rapid change in land use, (e.g., rapid deforestation) or by a rapid shift in the climate. Statistical hypothesis tests on randomness, regression analysis and hypothesis tests using the binomial distribution will be used to search for evidence of these aspects of non-randomness in the following sections.

8.2 The Kendall and Run tests of non-randomness

The Kendall (see Kendall 1975, Hirsch et al., 1981 and Taylor and Loftis, 1989) and Run tests (see Miller and Freund, 1965) are non-parametric (i.e., distribution free) hypothesis tests. The hypothesis is that the random series, such as a flood record, are observations of an independently and identically distributed random variable. This is the usual assumption made when estimating the flood-frequency distribution from a period of record. The Kendall test examines the alternate hypothesis that the series non-randomness is due to a trend. The run test examines serial dependence in the observed record as an alternative hypothesis. Serial dependence does not identify non-randomness; but its appearance in a sample might be caused by a trend or some other aspects of non-stationarity.

The Kendall test examines the frequency with which later values obtained in a record are smaller/larger on the average than should be expected in a random series. To examine this possibility, the following test statistic is computed:

$$
\begin{aligned}
z &= \frac{(S-1)}{\sqrt{V}} \qquad S>0 \\
z &= 0 \qquad\qquad S = 0 \qquad\qquad (8.1) \\
z &= \frac{(S+1)}{\sqrt{V}} \qquad S<0
\end{aligned}
$$

where z is approximately normally distributed when n > 30, where n is the number of observed flows, and:

$$S = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} \text{sign}\left[Q_j - Q_k\right] \qquad (8.2)$$

$$V = \frac{n(n-1)(2n+5) - \sum_{i=1}^{i=g} t_i\,(t_i - 1)(2t_i + 5)}{18} \qquad (8.3)$$

where Q_j, Q_k are observations of the random variable (e.g., flow), sign[] returns the value ±1 or zero depending on the sign of the difference in the bracket, g is the number of groups of ties, t is the number of ties in a group (e.g., if there are 5 ties of pairs of flows, and 3 ties of triplets of flows, then g=2, i=1, t_i=5, and i=2, t_i=3). The null hypothesis that the series is random is **not** rejected at some confidence level based on a one tailed test $z < z_\forall$, where ∀ is the significance level for rejecting the null hypothesis. A one tailed hypothesis test is applied in this case to test for a positive trend (i.e., a tendency for later flows Q_j to exceed earlier flows Q_k).

The Run test can be used to examine the extent to which there are more or less sequences of observations greater or less than some level, or runs, than would be expected from the observations of an independent random variable. The focus here is on a smaller magnitude of runs because the interest is in a positive serial correlation in the data that might mimic a positive trend or may be apparent statistically because of a positive trend. An increase on the average would tend to result in more consecutive values exceeding the median value than would be expected from random sampling variability. The test involves computing; 1) the number of runs, u defined corresponding to some level such as the median flow value; and 2) the number of values n_1 above, and n_2 below this level. The following statistic is used to test the null hypothesis that the number of runs do not deviate more than would be expected from a random process:

$$z = \frac{u - \bar{u}}{S_u} \qquad (8.4)$$

where z is distributed approximately normal when either n_1 or $n_2 > 20$, and, the expected number of runs is computed as:

$$\bar{u} = \frac{2n_1n_2}{n_1 + n_2} + 1 \tag{8.5}$$

and the corresponding variance as:

$$S_u^2 = \frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2\ (n_1+n_2-1)} \tag{8.6}$$

The test statistic, z, becomes more negative when a positive trend or episode causes increased flows with respect to some level. Consequently, a one-tail significance test is used to reject the null hypothesis when $z < z_{\alpha}$, where α is the significance level.

The Kendall and run tests were applied to selected gages throughout the study area. The results of the analysis are mixed as is shown in Tables 8.1 and 8.2. The tests do not agree completely on the existence of non-randomness at the gages examined. The run test does indicate the possibility of non-randomness more often then the Kendall test and at smaller significance levels (the smaller the significance level, the less chance that the deviation from the null hypothesis is due to random chance or sampling uncertainty). Consequently, the application of these tests provide some, but not conclusive, evidence of non-randomness being present in some of the study area flow records.

8.3 Regression analysis of trends

A linear regression analysis was performed to determine if a trend with time is present in the observed flow records. The regression analyzed has the form:

$$Q_i = a + bt_i + \varepsilon_i \tag{8.7}$$

where Q_i is the flow for year t_i, a is the regression constant, b is the slope indicating the trend, and $,_i$ is the regression residual. The regression constant and slope are obtained per the usual regression technique of minimizing the regression residuals summed over all observations.

The regression results shown in Table 8.3 provide both the correlation between flow and year, the Student's t statistic and corresponding significance level for the regression slope. The correlation between flow and time are small and do not indicate any strong linear association between time and flow magnitude. The t statistic can be used to test the null hypothesis of a zero (not significant) regression slope. This hypothesis is tested by determining if $t > t_{\forall}$, where $\forall$ is the significance level. Examination of table 8.3 reveals significance levels less than 10% at four of the seven gages tested, and only in one case, less than 5%. Consequently, the evidence for a trend (a positive regression slope) is not strongly supported by measures of statistical significance, nor does the correlation between flow and time make the regression significant from a prediction or engineering point of view.

Table 8.1: Results of Kendall Test for Non-Randomness in Annual Peak Series at Selected Stations

	significance level	10%	5%	1%
Station	[1]z [2]$z_\forall$	1.28	1.65	2.33
Anoka	1.04	[3]accept	accept	accept
Booneville	0.70	accept	accept	accept
Keokuk	1.27	accept	accept	accept
Scotland	1.56	reject	accept	accept
Sioux City	-2.44	accept	accept	accept
Saint Louis	1.28	reject/accept	accept	accept
Wapello	-1.59	accept	accept	accept

[1]run statistic for annual peaks using median flow value
[2]critical value for run statistic
[3]accept hypothesis of random series if $z<z_\forall$

Table 8.2: Result of Run Test for Non-Randomness in Annual Peak Series at Selected Stations

	significance level	10%	5%	1%
Station	[1]z [2]$z_{\forall}$	-1.28	-1.65	-2.33
Anoka	-1.26	[3]accept	accept	accept
Booneville	-1.02	accept	accept	accept
Keokuk	-2.05	reject	reject	accept
Scotland	-1.24	accept	accept	accept
Sioux City	-2.14	reject	reject	accept
Saint Louis	-2.05	reject	reject	accept
Wapello	-2.10	reject	reject	accept

[1]run statistic for annual peaks using median flow value
[2]critical value for run statistic
[3]accept hypothesis of random series if $z>z_{\forall}$

Table 8.3 Regression trend analysis for selected stations, annual peak versus year

Station	Record Length	correlation	[1]slope t-statistic	[2]significance level
Anoka	64	0.12	0.94	0.17
Booneville	99	0.08	0.74	0.23
Keokuk	118	0.14	1.61	0.06
Scotland	66	0.26	2.19	0.01
Sioux City	100	-0.17	-1.74	-----
Saint Louis	118	0.14	1.59	0.06
Wapello	92	0.03	0.28	0.06

[1]Student t statistic for regression slope
[2]Critical sginificance level for one-tail hypothesis test on regression slope, hypothesis is slope not greater than zero (i.e.. no positive trend with time)

8.4 Examination of expected exceedances from the binomial distribution

The period of record may not be indicative of the present day potential for flooding if a number of major floods occur in the very recent past. The apparent increase in flooding can be tested using the binomial distribution. In a period n, the expected number of exceedances of a flow level with exceedance probability p, is np. The variance associated with this estimate is $s_p^2=p(1-p)/n$. The statistical significance of the difference between observed and expected exceedances can be examined by the following test statistic:

$$z = \frac{(X/n) - p}{\sqrt{p(1-p)/n}} = \frac{(X/n) - p}{s_p} \qquad (8.8)$$

where X is the number of exceedances above some predefined flow level in period n. The null hypothesis, the number of exceedances in n years of a flow level with exceedance probability p is not different than expected from normal sampling variability, is **not** rejected if $z < z_{\forall}$, where $\forall$ is the significance level. In applying this sample estimate, p is used in place of the population value. The statistic employed is reasonable, but could be improved upon by finding a statistic that directly involves the sample estimate of p.

This hypothesis was examined for the unregulated flows recorded at the study area gages as is shown in Table 8.4. The exceedance probability was computed for the 5^{th} largest flow in the record from the estimated log-Pearson III distribution. The null hypothesis was tested for the most recent 25-years of record. The results indicate for most of the study area stations, the number of exceedances is not different than would be expected given a reasonable selection of the significance level ($z_{1.0\%}$ is 2.33). Interestingly, the number of exceedances is significantly different than expected for those sites in the lower reaches of the study area (below Hannibal). This result mirrors to some extent, the application of the Kendall and runs test (see Tables 8.1 and 8.2) where there was some evidence for non-randomness at St. Louis given significance levels as low as 5%. Consequently, the statistical evidence for non-randomness is not study-wide, but there is some evidence for a localized area of non-randomness within the study.

8.5 Conclusion

The statistical analysis does not provide a great deal of evidence supporting a hypothesis of non-randomness in the Upper Mississippi study region as a whole. The degree of dependence between annual peak flows for the study area gages makes it difficult to assess the number of gages independently revealing some aspect of non-randomness. However, the study area is very large, and either land use changes or climatic variability may have caused non-homogeneity or non-stationarity in the unregulated flow record. In particular, those areas in the Upper Mississippi below Hannibal exhibit a statistically significant deviation from the number of exceedances expected to occur over the past 25 years.

The study performed herein was extended by Olsen and Stakhiv (1999) to more gages using the similar approaches for studying non-randomness. They concluded (pg. 99):

> *There is evidence that flood risk has changed over time for sites where the 1993 flood was the flood of record, particularly at and below Hannibal, Missouri. This increased flood risk challenges the traditional assumption that flood series are independent and identically distributed random variables. This raises concerns that flood risk during the planning period will be underestimated if the entire flood record is used as the basis of projection of future flood risk.*

and:

> *It is not clear how to accommodate the change in flood risk within traditional flood frequency analysis. In the absence of viable alternatives the use of traditional Bulletin 17B procedures are warranted until better methods are developed.*

Consequently, the application of the standard flood frequency techniques over the period is recommended despite the evidence for trends in the mean annual flow identified for the lower portion of the study area.

Table 8.4: Comparison of expected and observed exceedances over the past 25 years for unregulated flow stations

Location	years (1)	X (2)	N (3)	P (4)	X/N (5)	s_p (6)	z (7)
Sioux City, Mo Ri	100	2	25	.080	.080	.027	-.017
Omaha, Mo Ri	100	2	25	.096	.080	.029	-.541
Nebraska City, Mo Ri	100	2	25	.060	.080	.024	.862
St Joseph, Mo Ri	100	3	25	.078	.120	.027	1.587
*Kansas City, Mo Ri	100	1	25	.046	.040	.021	-.276
*Booneville, Mo Ri	100	1	25	.054	.040	.023	-.614
Hermann, Mo Ri	100	3	25	.058	.120	.023	2.632
Anoka, Miss Ri	65	1	25	.125	.040	.041	-2.064
St Paul, Miss Ri	130	1	25	.037	.040	.017	.155
Mankato, Minn Ri	93	1	25	.042	.040	.021	-.092
Winona, Miss Ri	110	1	25	.060	.040	.023	-.890
McGregor, Miss Ri	58	1	25	.070	.040	.033	-.889
Muscoda, Wisc Ri	62	3	25	.045	.120	.026	2.817
Dubuque, Miss Ri	118	2	25	.038	.080	.018	2.380
Clinton, Miss Ri	122	1	25	.038	.040	.017	.099
*Keokuk, Miss Ri	118	2	25	.044	.080	.019	1.909
*Hannibal, Miss Ri	118	5	25	.047	.200	.020	7.805
*Louisiana, Miss Ri	67	4	25	.075	.160	.032	2.637
Meredosia, Ill Ri	74	4	25	.058	.160	.027	3.780
*Alton, Miss Ri	68	5	25	.076	.200	.032	3.885
*St Louis, Miss Ri	135	4	25	.053	.160	.019	5.506
*Chester, Miss Ri	71	4	25	.069	.160	.030	3.047

(1)years of record
(2)number of values equal or exceeding the 5^{th} largest event in 25 years
(3)period for binomial experiment (X success in N trials)
(4)proportion of success or exceedances of 5^{th} largest event in N trials
(5)exceedance probability of 5^{th} largest event of record, also probability of success in binomial experiment
(6)standard deviation of estimated probability for 5ht largest event in period of record
(7)test statistic $z=(X/N - NP)/s_p$, standard normal deviate, at significance level $\forall$=1%, $z_{\forall}=2.33$
*1993 event of record

9. Conclusions

The analysis did not show any practically significant different predictions of flood quantile or exceedance estimates by the Bulletin 17B procedures and the other distribution-estimation combination methods tested. However, a particular test, forecast split sample, almost universally resulted in the selection of the log-Normal distribution-standard moment combination. This selection occurred because there is an apparent increase in the frequency of large floods over the later half of the study area period of record. The zero-skew log-Normal distribution is favored in this case over the other distributions which were estimated to have negative skews based on the earlier part of the record.

The apparent increased risk in flooding may be due to some trend or non-homogeneity in the flood record. Nevertheless, the significance of the trend is not great enough to recommend deviation from the standard frequency analysis assumption of stationary flood records. Consequently, the recommendation is to obtain flood quantile estimates using the Bulletin 17B guidelines together with the TAG and IAG recommendations for regionalizing and smoothing distribution moments.

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Appendix A. Summary of quantile and exceedance probability comparisons

A.1 Introduction

The purpose of this sections is to provide a summary of the sample statistics and results of the comparison study described in Section 5. Complete tabular results are provided for both quantile and exceedance comparisons.

The following abbreviations are used in the tables to signify the combined distribution and estimation pairs:

l-gamma	Gamma distribution with L-moments
l-GEV	Generalized Extreme Value distribution with L-moments
l-g.logist	Generalized Logistic distribution with L-moments
l-g.pareto	Generalize Pareto distribution with L-moments
l-gumbel	Gumbel distribution with L-moments
l-l.normal	log-Normal distribution with L-moments
l-wakeby	Wakeby distribution with L-moments
m-lpIII	log-Pearson III with Bulletin 17B procedure
m-l.normal	log-Normal distribution with standard moments
m-gumbel	Gumbel distribution with standard moments
c-l.normal	log-Normal with regression applied to data censored at 50% chance exceedance
c-gumbel	Gumbel with regression applied to data censored at 50% chance exceedance

A.2 Quantile Comparisons

The results are organized as follows:

Tables A.1 (unregulated flows) and A.7 (USGS data):

Log-Pearson III statistics from program FFA (HEC, 1992), an application of the Bulletin 17B guidelines. Statistics are provided for the full data and the four alternative methods for dividing the data in split sample testing.

Tables A.2-A.4 (unregulated flows) and A.8-A.10 (USGS data):

Distribution and estimation methods selected as best in terms of bias, relative error and mean square error in comparisons with the Cunnane plotting positions for selected exceedance probabilities.

Tables A.5 (unregulated flows) and A.6 (USGS data)

Distribution and estimation methods selected as best in terms of mean square error in comparisons with the Weibull plotting positions for selected exceedance probabilities.

Tables A.6 (unregulated flows) and A.12 (USGS data)

Distribution and estimation methods selected as best in terms of mean square error in comparison with the Hosking and Wallis plotting positions for selected exceedance probabilities.

Table A.13 (unregulated flows)

Average difference between distributions ranked best based on mean square error for each estimation methodology and the log-Pearson III distribution at the 1% chance event.

Table A.14 (unregulated flows)

A comparison of the Bulletin 17B log-Pearson type III predictions versus the log-normal distribution obtained by various estimation methods for the 1% chance flood.

Table A.1: Statistics of Bulletin 17B analysis unregulated flow data set, full period of record

STATION NUMBER / STATION NAME AND LOCATION / AREA SQ MI	YEARS RECD	YEARS SYST	YEARS HIST	MEAN LOG	STD DEV	SKEW ADOPT	SKEW COMP	SKEW GENRL	HIST EVENT	OUTLIER HI	OUTLIER LO	ZERO/ MSNG
Sioux City, Mo Ri ,DA 314600, 35120 sq mi	100	100	0	5.158	.196	-.52	-.522	-99.00	0	0	0	0
Omaha, Mo Ri ,DA 322820, 43340 sq mi	100	100	0	5.157	.186	-.47	-.471	-99.00	0	0	0	0 ..
Nebraska City, Mo Ri , DA 414420, 134940 sq mi	100	100	0	5.216	.187	-.49	-.494	-99.00	0	0	0	0
St Joseph, Mo Ri , DA 429340, 149860 sq mi	100	100	0	5.226	.151	.07	.070	-99.00	0	0	1	0
Kansas City, Mo Ri , DA 489162, 209862 sq mi	100	100	0	5.322	.184	-.02	-.018	-99.00	0	0	0	0
Booneville, Mo Ri ,DA 505710, 226230 sq mi	100	100	0	5.409	.186	-.10	-.097	-99.00	0	0	0	0
Hermann, Mo Ri ,DA 528200, 248720 sq mi	100	100	0	5.492	.196	.02	.025	-99.00	0	0	0	0
Anoka, Miss Ri ,DA 19600 sq mi	65	65	0	4.459	.219	-.37	-.374	-99.00	0	0	1	0
St Paul, Miss Ri ,DA 36800 sq mi	130	130	0	4.576	.269	-.26	-.256	-99.00	0	0	0	0
Mankato, Minn Ri , DA 14900 sq mi	93	93	0	4.182	.349	-.40	-.401	-99.00	0	0	1	0
Winona, Miss Ri , DA 59200 sq mi	110	110	0	4.934	.191	-.11	-.114	-99.00	0	0	0	0
McGregor, Miss Ri , DA 67500 sq mi	58	58	0	5.031	.162	-.05	-.047	-99.00	0	0	0	0
Muscoda, Wisc Ri , DA 10400 sq mi	62	62	0	4.648	.159	-.72	-.724	-99.00	0	0	1	0
Dubuque, Miss Ri , DA 82000 sq mi	118	118	0	5.095	.150	-.23	-.233	-99.00	0	0	3	0
Clinton, Miss Ri , DA 85600 sq mi	122	122	0	5.129	.147	-.24	-.236	-99.00	0	0	1	0
Keokuk, Miss Ri , DA 119000 sq mi	118	118	0	5.253	.149	-.39	-.387	-99.00	0	0	1	0
Hannibal, Miss Ri , DA 137000 sq mi	118	118	0	5.297	.166	-.36	-.358	-99.00	0	0	1	0
Louisiana, Miss Ri , DA 141000 sq mi	67	67	0	5.368	.147	-.11	-.114	-99.00	0	0	0	0
Meredosia, Ill Ri , DA 26030	74	74	0	4.790	.172	-.42	-.420	-99.00	0	0	1	0
Alton, Miss Ri ,DA 171300 sq mi	68	68	0	5.441	.142	.10	.099	-99.00	0	0	1	0
St Louis, Miss Ri ,DA 697000, 417520 sq mi	135	135	0	5.691	.151	-.17	-.170	-99.00	0	0	1	0
Chester, Miss Ri ,DA 708600, 429120 sq mi	71	71	0	5.725	.168	-.35	-.353	-99.00	0	0	1	0
Thebes, Miss Ri ,DA 713200, 433720 sq mi	64	64	0	5.734	.163	-.39	-.390	-99.00	0	0	1	0

Table A.1: Statistics of Bulletin 17B analysis unregulated flow data set, first half unregulated data, (continued)

STATION NUMBER / STATION NAME AND LOCATION / AREA SQ MI	YEARS RECD	YEARS SYST	YEARS HIST	MEAN LOG	STD DEV	SKEW ADOPT	SKEW COMP	SKEW GENRL	HIST EVENT	OUTLIER HI	OUTLIER LO	ZERO/ MSNG
Sioux City, Mo Ri ,DA 314600, 35120 sq mi	50	50	0	5.187	.152	-1.00	-.998	-99.00	0	0	1	0
Omaha, Mo Ri ,DA 322820, 43340 sq mi	50	50	0	5.155	.156	-.96	-.961	-99.00	0	0	0	0
Nebraska City, Mo Ri , DA 414420, 134940 sq mi	50	50	0	5.226	.152	-.46	-.456	-99.00	0	0	1	0
St Joseph, Mo Ri , DA 429340, 149860 sq mi	50	50	0	5.185	.135	-.63	-.628	-99.00	0	0	0	0
Kansas City, Mo Ri , DA 489162, 209862 sq mi	50	50	0	5.318	.183	-.40	-.399	-99.00	0	0	0	0
Booneville, Mo Ri ,DA 505710, 226230 sq mi	50	50	0	5.396	.196	-.43	-.426	-99.00	0	0	0	0
Hermann, Mo Ri ,DA 528200, 248720 sq mi	50	50	0	5.468	.196	-.26	-.258	-99.00	0	0	0	0
Anoka, Miss Ri ,DA 19600 sq mi	32	32	0	4.417	.250	-.75	-.753	-99.00	0	0	0	0
St Paul, Miss Ri ,DA 36800 sq mi	65	65	0	4.521	.276	-.26	-.260	-99.00	0	0	0	0
Mankato, Minn Ri , DA 14900 sq mi	46	46	0	4.064	.365	-.67	-.674	-99.00	0	0	1	0
Winona, Miss Ri , DA 59200 sq mi	55	55	0	4.889	.187	-.29	-.291	-99.00	0	0	0	0

McGregor, Miss Ri	, DA 67500 sq mi	29	29	0	5.000	.166	.63	.634	-99.00	0	0	0	0
Muscoda, Wisc Ri	, DA 10400 sq mi	31	31	0	4.664	.129	-.34	-.336	-99.00	0	0	1	0
Dubuque, Miss Ri	, DA 82000 sq mi	59	59	0	5.023	.172	-.95	-.946	-99.00	0	0	1	0
Clinton, Miss Ri	, DA 85600 sq mi	61	61	0	5.117	.146	-.05	-.050	-99.00	0	0	1	0
Keokuk, Miss Ri	, DA 119000 sq mi	59	59	0	5.226	.145	-.57	-.566	-99.00	0	0	1	0
Hannibal, Miss Ri	, DA 137000 sq mi	59	59	0	5.227	.154	-.75	-.749	-99.00	0	0	1	0
Louisiana, Miss Ri	, DA 141000 sq mi	33	33	0	5.323	.140	-.40	-.397	-99.00	0	0	0	0
Meredosia, Ill Ri	, DA 26030	37	37	0	4.728	.203	-.58	-.584	-99.00	0	0	0	0
Alton, Miss Ri	,DA 171300 sq mi	34	34	0	5.387	.126	.15	.153	-99.00	0	0	1	0
St Louis, Miss Ri	,DA 697000, 417520 sq mi	67	67	0	5.680	.138	-.18	-.178	-99.00	0	0	0	0
Chester, Miss Ri	,DA 708600, 429120 sq mi	35	35	0	5.689	.181	-.52	-.522	-99.00	0	0	1	0
Thebes, Miss Ri	,DA 713200, 433720 sq mi	32	32	0	5.689	.175	-.59	-.586	-99.00	0	0	1	0

Table A.1: Statistics of Bulletin 17B analysis unregulated flow data set (continued), unregulated data, second half

STATION NUMBER / STATION NAME AND LOCATION	AREA SQ MI	YEARS RECD	YEARS SYST	YEARS HIST	MEAN LOG	STD DEV	SKEW ADOPT	SKEW COMP	SKEW GENRL	HIST EVENT	OUTLIER HI	OUTLIER LO	ZERO/ MSNG
Sioux City, Mo Ri ,DA 314600, 35120 sq mi		50	50	0	5.134	.224	-.13	-.131	-99.00	0	0	0	0
Omaha, Mo Ri ,DA 322820, 43340 sq mi		50	50	0	5.160	.214	-.29	-.287	-99.00	0	0	0	0 ..
Nebraska City, Mo Ri , DA 414420, 134940 sq mi		50	50	0	5.211	.209	-.37	-.367	-99.00	0	0	0	0
St Joseph, Mo Ri , DA 429340, 149860 sq mi		50	50	0	5.267	.158	.25	.247	-99.00	0	0	1	0
Kansas City, Mo Ri , DA 489162, 209862 sq mi		50	50	0	5.326	.186	.34	.342	-99.00	0	0	0	0
Booneville, Mo Ri ,DA 505710, 226230 sq mi		50	50	0	5.422	.177	.40	.400	-99.00	0	0	0	0
Hermann, Mo Ri ,DA 528200, 248720 sq mi		50	50	0	5.516	.195	.32	.316	-99.00	0	0	0	0
Anoka, Miss Ri ,DA 19600 sq mi		33	33	0	4.485	.217	-.27	-.267	-99.00	0	0	0	0
St Paul, Miss Ri ,DA 36800 sq mi		65	65	0	4.639	.234	.22	.218	-99.00	0	0	1	0
Mankato, Minn Ri , DA 14900 sq mi		47	47	0	4.292	.310	.15	.147	-99.00	0	0	0	0
Winona, Miss Ri , DA 59200 sq mi		55	55	0	4.980	.185	.05	.055	-99.00	0	0	0	0
McGregor, Miss Ri , DA 67500 sq mi		29	29	0	5.073	.132	-.22	-.218	-99.00	0	0	1	0
Muscoda, Wisc Ri , DA 10400 sq mi		31	31	0	4.635	.183	-.70	-.703	-99.00	0	0	0	0
Dubuque, Miss Ri , DA 82000 sq mi		59	59	0	5.150	.140	-.31	-.309	-99.00	0	0	0	0
Clinton, Miss Ri , DA 85600 sq mi		61	61	0	5.146	.141	-.20	-.203	-99.00	0	0	1	0
Keokuk, Miss Ri , DA 119000 sq mi		59	59	0	5.277	.153	-.37	-.373	-99.00	0	0	0	0
Hannibal, Miss Ri , DA 137000 sq mi		59	59	0	5.365	.156	-.35	-.346	-99.00	0	0	0	0
Louisiana, Miss Ri , DA 141000 sq mi		34	34	0	5.412	.143	.08	.077	-99.00	0	0	0	0
Meredosia, Ill Ri , DA 26030		37	37	0	4.841	.147	-.23	-.227	-99.00	0	0	0	0
Alton, Miss Ri ,DA 171300 sq mi		34	34	0	5.494	.143	-.20	-.201	-99.00	0	0	0	0
St Louis, Miss Ri ,DA 697000, 417520 sq mi		68	68	0	5.701	.167	-.27	-.266	-99.00	0	0	1	0
Chester, Miss Ri ,DA 708600, 429120 sq mi		36	36	0	5.757	.155	-.02	-.024	-99.00	0	0	0	0
Thebes, Miss Ri ,DA 713200, 433720 sq mi		32	32	0	5.777	.147	.05	.046	-99.00	0	0	0	0

Table A.1: Statistics of Bulletin 17B analysis Unregulated Flow Data Set (continued), unregulated data, alternate values, first, third, fifth, etc., values in record

STATION NUMBER / STATION NAME AND LOCATION	AREA SQ MI	YEARS RECD	YEARS SYST	YEARS HIST	MEAN LOG	STD DEV	SKEW ADOPT	SKEW COMP	SKEW GENRL	HIST EVENT	OUTLIER HI	OUTLIER LO	ZERO/ MSNG
Sioux City, Mo Ri ,DA 314600, 35120 sq mi		50	50	0	5.143	.207	-.19	-.189	-99.00	0	0	0	0
Omaha, Mo Ri ,DA 322820, 43340 sq mi		50	50	0	5.142	.195	-.02	-.024	-99.00	0	0	0	0 ..
Nebraska City, Mo Ri , DA 414420, 134940 sq mi		50	50	0	5.202	.191	-.03	-.029	-99.00	0	0	0	0
St Joseph, Mo Ri , DA 429340, 149860 sq mi		50	50	0	5.204	.156	.56	.558	-99.00	0	0	1	0
Kansas City, Mo Ri , DA 489162, 209862 sq mi		50	50	0	5.277	.174	-.39	-.394	-99.00	0	0	0	0
Booneville, Mo Ri ,DA 505710, 226230 sq mi		50	50	0	5.366	.178	-.38	-.375	-99.00	0	0	0	0
Hermann, Mo Ri ,DA 528200, 248720 sq mi		50	50	0	5.442	.178	-.46	-.459	-99.00	0	0	0	0
Anoka, Miss Ri ,DA 19600 sq mi		33	33	0	4.454	.240	-.25	-.247	-99.00	0	0	0	0
St Paul, Miss Ri ,DA 36800 sq mi		65	65	0	4.578	.204	-.19	-.188	-99.00	0	0	1	0
Mankato, Minn Ri , DA 14900 sq mi		47	47	0	4.225	.393	-.70	-.701	-99.00	0	0	1	0
Winona, Miss Ri , DA 59200 sq mi		55	55	0	4.916	.174	-.28	-.284	-99.00	0	0	0	0

McGregor, Miss Ri	, DA 67500 sq mi	29	29	0	5.045	.180	-.07	-.072	-99.00	0	0	0	0
Muscoda, Wisc Ri	, DA 10400 sq mi	31	31	0	4.628	.196	-.93	-.934	-99.00	0	0	0	0
Dubuque, Miss Ri	, DA 82000 sq mi	59	59	0	5.076	.190	-.82	-.818	-99.00	0	0	1	0
Clinton, Miss Ri	, DA 85600 sq mi	61	61	0	5.128	.152	-.26	-.259	-99.00	0	0	1	0
Keokuk, Miss Ri	, DA 119000 sq mi	59	59	0	5.246	.156	-.34	-.336	-99.00	0	0	1	0
Hannibal, Miss Ri	, DA 137000 sq mi	59	59	0	5.299	.179	-.30	-.302	-99.00	0	0	1	0
Louisiana, Miss Ri	, DA 141000 sq mi	34	34	0	5.352	.164	-.15	-.149	-99.00	0	0	0	0
Meredosia, Ill Ri	, DA 26030	37	37	0	4.769	.178	-.20	-.199	-99.00	0	0	0	0
Alton, Miss Ri	,DA 171300 sq mi	34	34	0	5.431	.150	.14	.143	-99.00	0	0	1	0
St Louis, Miss Ri	,DA 697000, 417520 sq mi	68	68	0	5.701	.155	-.34	-.340	-99.00	0	0	0	0
Chester, Miss Ri	,DA 708600, 429120 sq mi	36	36	0	5.688	.158	-.87	-.871	-99.00	0	0	1	0
Thebes, Miss Ri	,DA 713200, 433720 sq mi	32	32	0	5.768	.164	-.04	-.038	-99.00	0	0	0	0

Table A.1: Statistics of Bulletin 17B analysis unregulated flow data set (continued), unregulated data, alternate values, second, fourth, sixth, etc., flow values in record

STATION NUMBER / STATION NAME AND LOCATION, AREA SQ MI	YEARS RECD	YEARS SYST	YEARS HIST	MEAN LOG	STD DEV	SKEW ADOPT	SKEW COMP	SKEW GENRL	HIST EVENT	OUTLIER HI	OUTLIER LO	ZERO/ MSNG
Sioux City, Mo Ri , 314600, 35120 sq mi	50	50	0	5.173	.185	-.96	-.955	-99.00	0	0	0	0
Omaha, Mo Ri ,DA 322820, 43340 sq mi	50	50	0	5.173	.177	-1.04	-1.038	-99.00	0	0	0	0 ..
Nebraska City, Mo Ri , DA 414420, 134940 sq mi	50	50	0	5.235	.173	-.88	-.881	-99.00	0	0	1	0
St Joseph, Mo Ri , DA 429340, 149860 sq mi	50	50	0	5.253	.136	-.22	-.217	-99.00	0	0	1	0
Kansas City, Mo Ri , DA 489162, 209862 sq mi	50	50	0	5.367	.184	.22	.220	-99.00	0	0	0	0
Booneville, Mo Ri ,DA 505710, 226230 sq mi	50	50	0	5.453	.186	.06	.058	-99.00	0	0	0	0
Hermann, Mo Ri ,DA 528200, 248720 sq mi	50	50	0	5.541	.201	.20	.203	-99.00	0	0	0	0
Anoka, Miss Ri ,DA 19600 sq mi	32	32	0	4.463	.204	-.63	-.634	-99.00	0	0	1	0
St Paul, Miss Ri ,DA 36800 sq mi	65	65	0	4.582	.310	-.17	-.171	-99.00	0	0	0	0
Mankato, Minn Ri , DA 14900 sq mi	46	46	0	4.135	.307	-.22	-.220	-99.00	0	0	0	0
Winona, Miss Ri , DA 59200 sq mi	55	55	0	4.953	.206	-.10	-.095	-99.00	0	0	0	0
McGregor, Miss Ri , DA 67500 sq mi	29	29	0	5.017	.143	-.18	-.184	-99.00	0	0	0	0
Muscoda, Wisc Ri , DA 10400 sq mi	31	31	0	4.658	.140	-.53	-.529	-99.00	0	0	0	0
Dubuque, Miss Ri , DA 82000 sq mi	59	59	0	5.097	.143	-.43	-.428	-99.00	0	0	0	0
Clinton, Miss Ri , DA 85600 sq mi	61	61	0	5.129	.146	-.26	-.264	-99.00	0	0	0	0
Keokuk, Miss Ri , DA 119000 sq mi	59	59	0	5.257	.146	-.48	-.483	-99.00	0	0	0	0
Hannibal, Miss Ri , DA 137000 sq mi	59	59	0	5.300	.145	-.18	-.175	-99.00	0	0	1	0
Louisiana, Miss Ri , DA 141000 sq mi	33	33	0	5.385	.128	.23	.231	-99.00	0	0	0	0
Meredosia, Ill Ri , DA 26030	37	37	0	4.810	.169	-.74	-.744	-99.00	0	0	1	0
Alton, Miss Ri ,DA 171300 sq mi	34	34	0	5.450	.139	.03	.028	-99.00	0	0	0	0
St Louis, Miss Ri ,DA 697000, 417520 sq mi	67	67	0	5.680	.151	-.05	-.046	-99.00	0	0	1	0
Chester, Miss Ri ,DA 708600, 429120 sq mi	35	35	0	5.760	.179	-.28	-.282	-99.00	0	0	0	0
Thebes, Miss Ri ,DA 713200, 433720 sq mi	32	32	0	5.696	.165	-.99	-.995	-99.00	0	0	1	0

Table A.2: Distributions with minimum bias for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*c-l.normal	*c-l.normal	*l-g.logist
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*m-l.normal	**l-gumbel	-----
alternate even split	*l-g.logist	**m-l.pIII	-----

*Based on 23 Stations
**Based on 22 Stations

Table A.3: Distributions with minimum absolute relative error for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*l-wakeby	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-wakeby	**l-gamma	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*Based on 23 Stations
**Based on 22 Stations

Table A.4: Distributions with minimum mean square error for selected exceedance probabilities, Cunnane plotting position, unregulated data

Test	0.100	0.020	0.010
Period of record	*l-wakeby	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-l.normal	**l-gumbel	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*Based on 23 Stations
**Based on 22 Stations

Table A.5: Distributions with minimum mean square error for selected exceedance probabilities, Weibull plotting positions, unregulated data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*l-wakeby	*c-gumbel	*m-l.normal
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-l.normal	**l-l.normal	-----
alternate even split	*m-l.pIII	**m-l.normal	-----

*Based on 23 Stations
**Based on 14 Stations
[1]Exceedance probability

Table A.6: Distributions with minimum mean square error for selected exceedance probabilities, Hosking and Wallis (1997) plotting position, unregulated data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*l-wakeby	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-g.pareto	-----
alternate odd split	*l-l.normal	**l-gumbel	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*Based on 23 Stations
**Based on 20 Stations
[1]Exceedance probability

Table A.7: Statistics of Bulletin 17B analysis, USGS data full period of record

STATION NUMBER	STATION NAME AND LOCATION	AREA SQ MI	....YEARS..... RECD	SYST	HIST	MEAN LOG	STD DEV	.......SKEW........ ADOPT	COMP	GENRL	HIST EVENT	OUTLIER HI	LO	ZERO/ MSNG
	Conesville,IA,Cedar Ri,DA sq mi	7785	55	55	0	4.406	.290	-.61	-.607	-99.00	0	0	0	0
	Wapello, IA, Iowa Ri ,DA sq mi	12499	92	92	0	4.548	.253	-.69	-.692	-99.00	0	0	1	0
	Augusta, IA, Skunk Ri ,DA sq mi	4303	80	80	0	4.304	.224	-.48	-.484	-99.00	0	0	1	0
	Fort Dodge,IA,Des Moines Ri,DA sq mi	4190	62	62	0	4.013	.290	-.20	-.205	-99.00	0	0	0	0
	Stratford,IA,Des Moines Ri, DA sq mi	5452	27	27	0	4.180	.282	-.69	-.690	-99.00	0	0	0	0
	Van Meter,IA, Raccoon Ri, DA sq mi	3441	80	80	0	4.154	.266	-.05	-.052	-99.00	0	0	1	0
	Scotland, SD,James Ri , DA sq mi	20653	66	66	0	3.339	.471	.11	.110	-99.00	0	0	1	0
	Brookings,SD,Big Sioux Ri,DA sq mi	3898	41	41	0	3.364	.533	-.29	-.293	-99.00	0	0	0	0
	Waterloo,NE,Elkhorn Ri, DA sq mi	6900	76	76	0	4.090	.333	.12	.122	-99.00	0	0	0	0
	Beatrice,NE,Big Blue Ri, DA sq mi	3901	91	91	0	3.935	.425	-.43	-.434	-99.00	0	0	0	0
	Sumner,MO,Grand Ri ,DA sq mi	6880	72	72	0	4.730	.214	-.07	-.075	-99.00	0	0	2	0

Table A.7: Statistics of Bulletin 17B analysis, USGS data first half (continued)

STATION NUMBER	STATION NAME AND LOCATION	AREA SQ MI	....YEARS..... RECD	SYST	HIST	MEAN LOG	STD DEV	.......SKEW........ ADOPT	COMP	GENRL	HIST EVENT	OUTLIER HI	LO	ZERO/ MSNG
	Conesville,IA, Cedar Ri,DA sq mi	7785	27	27	0	4.390	.343	-.58	-.578	-99.00	0	0	0	0
	Wapello, IA, Iowa Ri ,DA sq mi	12499	46	46	0	4.553	.242	-1.02	-1.020	-99.00	0	0	1	0
	Augusta, IA, Skunk Ri , DA sq mi	4303	40	40	0	4.310	.181	-.43	-.431	-99.00	0	0	1	0
	Fort Dodge, IA, Des Moines Ri, DA sq mi	4190	31	31	0	3.987	.280	.17	.175	-99.00	0	0	0	0
	Stratford, IA, Des Moines Ri, DA sq mi	5452	13	13	0	4.105	.281	-.83	-.831	-99.00	0	0	0	0
	Van Meter, IA, Raccoon Ri, DA sq mi	3441	40	40	0	4.116	.234	.20	.199	-99.00	0	0	1	0
	Scotland, SD, James Ri , DA sq mi	20653	33	33	0	3.270	.438	-.26	-.255	-99.00	0	0	0	0
	Brookings, SD, Big Sioux RI, DA sq mi	3898	20	20	0	3.332	.586	-.08	-.081	-99.00	0	0	0	0
	Waterloo, NE, Elkhorn RI , DA sq mi	6900	38	38	0	3.968	.327	.67	.666	-99.00	0	0	0	0
	Beatrice, NE, Big Blue RI, DA sq mi	3901	45	45	0	3.837	.451	-.34	-.336	-99.00	0	0	0	0
	Sumner, MO, Grand RI, sq mi	6880	36	36	0	4.650	.288	-.67	-.671	-99.00	0	0	0	0

Table A.7: Statistics of Bulletin 17B analysis, USGS data second half (continued)

STATION NUMBER	STATION NAME AND LOCATION	AREA SQ MI	....YEARS..... RECD	SYST	HIST	MEAN LOG	STD DEV	.......SKEW........ ADOPT	COMP	GENRL	HIST EVENT	OUTLIER HI	LO	ZERO/ MSNG
	Conesville,IA, Cedar Ri,DA sq mi	7785	28	28	0	4.435	.209	.05	.049	-99.00	0	0	1	0
	Wapello, IA, Iowa Ri ,DA sq mi	12499	46	46	0	4.549	.253	-.28	-.281	-99.00	0	0	1	0
	Augusta, IA, Skunk Ri , DA sq mi	4303	40	40	0	4.298	.263	-.45	-.453	-99.00	0	0	0	0
	Fort Dodge, IA, Des Moines Ri, DA sq mi	4190	31	31	0	4.039	.302	-.56	-.559	-99.00	0	0	0	0
	Stratford, IA, Des Moines Ri, DA sq mi	5452	14	14	0	4.249	.275	-.76	-.760	-99.00	0	0	0	0

Van Meter, IA, Raccoon Ri, DA sq mi 3441	40	40	0	4.189	.298	-.35	-.347	-99.00	0	0	0	0
Scotland, SD, James Ri , DA sq mi 20653	33	33	0	3.405	.506	.21	.212	-99.00	0	0	1	0
Brookings, SD, Big Sioux RI, DA sq mi 3898	21	21	0	3.430	.417	.07	.066	-99.00	0	0	1	0
Waterloo, NE, Elkhorn RI , DA sq mi 6900	38	38	0	4.212	.295	-.24	-.244	-99.00	0	0	0	0
Beatrice, NE, Big Blue RI, DA sq mi 3901	46	46	0	4.030	.378	-.37	-.368	-99.00	0	0	0	0
Sumner, MO, Grand RI, sq mi 6880	36	36	0	4.778	.190	-.13	-.128	-99.00	0	0	0	0

Table A.7: Statistics of Bulletin 17B analysis USGS alternate data, first, third, fifth, etc., flow values in record (continued)

STATION NUMBER	STATION NAME AND LOCATION	AREA SQ MI	....YEARS..... RECD	SYST	HIST	MEAN LOG	STD DEV	.......SKEW........ ADOPT	COMP	GENRL	HIST EVENT	OUTLIER HI	LO	ZERO/ MSNG
	Conesville,IA, Cedar Ri,DA sq mi	7785	28	28	0	4.363	.295	-.89	-.892	-99.00	0	0	0	0
	Wapello, IA, Iowa Ri ,DA sq mi	12499	46	46	0	4.576	.247	-.56	-.558	-99.00	0	0	1	0
	Augusta, IA, Skunk Ri , DA sq mi	4303	40	40	0	4.293	.215	-.10	-.095	-99.00	0	0	0	0
	Fort Dodge, IA, Des Moines Ri, DA sq mi	4190	31	31	0	4.047	.297	-.20	-.204	-99.00	0	0	0	0
	Stratford, IA, Des Moines Ri, DA sq mi	5452	14	14	0	4.114	.263	-.72	-.723	-99.00	0	0	0	0
	Van Meter, IA, Raccoon Ri, DA sq mi	3441	40	40	0	4.176	.258	.21	.208	-99.00	0	0	0	0
	Scotland, SD, James Ri , DA sq mi	20653	33	33	0	3.297	.450	-.28	-.285	-99.00	0	0	1	0
	Brookings, SD, Big Sioux RI, DA sq mi	3898	21	21	0	3.359	.531	-.42	-.418	-99.00	0	0	0	0
	Waterloo, NE, Elkhorn RI , DA sq mi	6900	38	38	0	4.041	.318	-.16	-.161	-99.00	0	0	0	0
	Beatrice, NE, Big Blue RI, DA sq mi	3901	46	46	0	3.884	.447	-.34	-.337	-99.00	0	0	0	0
	Sumner, MO, Grand RI, sq mi	6880	36	36	0	4.706	.276	-.87	-.873	-99.00	0	0	1	0

Table A.7: Statistics of Bulletin 17B analysis USGS alternate data, second, fourth, sixth, etc., flow values in record (continued)

STATION NUMBER	STATION NAME AND LOCATION	AREA SQ MI	....YEARS..... RECD	SYST	HIST	MEAN LOG	STD DEV	.......SKEW........ ADOPT	COMP	GENRL	HIST EVENT	OUTLIER HI	LO	ZERO/ MSNG
	Conesville,IA, Cedar Ri,DA sq mi	7785	27	27	0	4.451	.284	-.32	-.320	-99.00	0	0	0	0
	Wapello, IA, Iowa Ri ,DA sq mi	12499	46	46	0	4.520	.260	-.82	-.823	-99.00	0	0	0	0
	Augusta, IA, Skunk Ri , DA sq mi	4303	40	40	0	4.313	.242	-.89	-.894	-99.00	0	0	1	0
	Fort Dodge, IA, Des Moines Ri, DA sq mi	4190	31	31	0	3.979	.284	-.26	-.263	-99.00	0	0	0	0
	Stratford, IA, Des Moines Ri, DA sq mi	5452	13	13	0	4.250	.295	-1.04	-1.042	-99.00	0	0	0	0
	Van Meter, IA, Raccoon Ri, DA sq mi	3441	40	40	0	4.116	.306	-.54	-.544	-99.00	0	0	0	0
	Scotland, SD, James Ri , DA sq mi	20653	33	33	0	3.376	.503	.30	.301	-99.00	0	0	0	0
	Brookings, SD, Big Sioux RI, DA sq mi	3898	20	20	0	3.368	.548	-.20	-.198	-99.00	0	0	0	0
	Waterloo, NE, Elkhorn RI , DA sq mi	6900	38	38	0	4.139	.344	.29	.294	-99.00	0	0	0	0
	Beatrice, NE, Big Blue RI, DA sq mi	3901	45	45	0	3.987	.399	-.51	-.506	-99.00	0	0	0	0
	Sumner, MO, Grand RI, sq mi	6880	36	36	0	4.733	.196	.33	.330	-99.00	0	0	0	0

Table A.8: Distributions with minimum bias for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
*Period of record	*c-gumbel	*l-wakeby	*m-l.pIII
**forecast split record	**m-l.normal	**m-l.normal	-----
**hindcast split record	**l-g.logist	**l-gumbel	-----
**alternate odd split	**m-l.normal	**c-l.normal	-----
**alternate even split	**l-g.logist	**c-l.normal	-----

*Based on 11 stations
**Based on 8 stations
[1]Exceedance probability

Table A.9: Distributions with minimum relative error for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*c-gumbel	*l-wakeby	*l-g.logist
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-gumbel	-----
alternate odd split	*m-l.normal	**l-gamma	-----
alternate even split	*l-g.logist	**l-gumbel	-----

*Based on 11 stations
**Based on 8 stations
[1]Exceedance probability

Table A.10: Distributions with minimum mean square error for selected exceedance probabilities, Cunnane plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*l-gamma	*l-wakeby	*l-wakeby
forecast split record	*m-l.normal	**l-l.normal	-----
hindcast split record	*l-g.logist	**l-gumbel	-----
alternate odd split	*l-l.normal	**c-l.normal	-----
alternate even split	*l-g.logist	**l-gumbel	-----

*11 stations
**8 stations
[1]Exceedance probability

Table A.11: Distributions with minimum mean square error for selected exceedance probabilities, Weibull plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*c-l.normal	*c-gumbel	-----
forecast split record	*c-gumbel	-----	-----
hindcast split record	*l-g.logist	-----	-----
alternate odd split	*c-gumbel	-----	-----
alternate even split	*l-GEV	-----	-----

*11 Stations
[1]Exceedance probability

Table A.12: Distributions with minimum mean square error for selected exceedance probabilities, Hosking and Wallis plotting position, USGS data

Test	[1]0.100	[1]0.020	[1]0.010
Period of record	*l-gamma	*l-wakeby	*l-g.logist
forecast split record	*m-l.normal	**m-l.normal	-----
hindcast split record	*l-g.logist	**l-gumbel	-----
alternate odd split	*l-l.normal	**l-l.normal	-----
alternate even split	*l-g.logist	**m-l.normal	-----

*11 stations
**7 stations
[1]Exceedance probability

Table A.13: Comparison of average difference between log-Pearson III and best distribution and estimation procedure combination for the 1% chance event, mean square error selection criteria at 2% chance exceedance, Cunnane Plotting Position Formula

test type	(1)	(2)	(3)	(4)	(5)	(6)
Period of record	l-wakeby*	.31	m-gumbel	2.11	c-gumbel	1.76
forecast split record	l-l.normal	8.67	m-l.normal*	9.94	c-l.normal	.80
hindcast split record	l-g.pareto*	-7.32	m-gumbel	2.11	c-l.normal	.80
alternate odd split	l-gumbel*	2.39	m-gumbel	2.11	c-l.normal	.80
alternate even split	l-l.normal	8.67	m-l.normal*	9.94	c-l.normal	.80

(1)Best distribution based on mean square error, L-moment estimation

(2)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

(3)Best distribution based on mean square error, standard moment estimation

(4)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

(5)Best distribution based on mean square error, regression application to censored data

(6)Average relative difference between log-Pearson III and distribution prediction for 1% chance flow

Table A.14: %Difference between log-Pearson III and normal distributions for 1% Chance Event

Location	m-l.pIII[1]	l-l.normal[2]	m-l.normal[3]	c-l.normal[4]
Sioux City, Mo Ri	345100	16.23	19.10	.87
Omaha, Mo Ri	334700	14.05	16.38	.61
Nebraska City, Mo Ri	382300	15.89	16.99	.76
St Joseph, Mo Ri	384600	.13	1.76	-3.02
Kansas City, Mo Ri	559200	-1.62	.38	2.98
Booneville, Mo Ri	673600	3.27	3.45	2.74
Hermann, Mo Ri	894400	-.03	-1.11	-1.30
Anoka, Miss Ri	80850	20.84	22.89	1.91
St Paul, Miss Ri	141500	11.68	12.21	1.64
Mankato, Minn Ri	77620	34.42	40.80	13.56
Winona, Miss Ri	230300	4.47	3.66	.17
McGregor, Miss Ri	252500	.69	1.11	5.60
Muscoda, Wisc Ri	85580	24.76	27.50	5.08
Dubuque, Miss Ri	261900	13.78	17.89	2.91
Clinton, Miss Ri	278800	9.78	10.27	.69
Keokuk, Miss Ri	360400	12.81	14.60	3.82
Hannibal, Miss Ri	435600	16.89	23.06	3.31
Louisiana, Miss Ri	498400	4.26	3.19	.88
Meredosia, Ill Ri	136900	18.47	19.59	.88
Alton, Miss Ri	604800	2.83	2.31	-1.68
St Louis, Miss Ri	1055000	8.49	8.83	.10
Chester, Miss Ri	1180000	16.57	17.33	-.17
Thebes, Miss Ri	1164000	17.93	19.20	.69
average		11.59	13.10	1.87

[1]Prediction of 1% chance event using LP III distribution
[2]Percent difference prediction log-Normal estimated with L-moments
[3]Percent difference prediction log-Normal estimated with standard moments
[4]Percent difference prediction log-Normal estimated using censored data

A.3 Exceedances

Comparisons were made for both the full data set (see Table 2.1) and for a consistent period of record (1933-1996). A consistent period of record was considered to obtain comparisons for plotting positions estimates based on the same record length; and consequently, the same estimation accuracy (the error in estimating the exceedance probability of the top ranked quantile depends on the record length).

The results of the investigation are presented as follows:

Aggregate bias for entire record (alternate split sample test)

Table A.15-A.18 provides a comparison of the observed bias of all the methods at the 10%, 2% and 1% chance exceedance events for the alternate split record testing. The method of moments log-normal and log-Pearson III use expected probability. This result is typical of other comparisons made in that no method is clearly superior over all.

Accuracy measures, full period of record (alternate split sample test)

Tables A.19-A.20 provides the results for all distributions in the comparisons for 23 stations, full period of record in the alternate split sample analysis. There is no clear choice, although L-moment estimation tends to dominate.

Summary results, full period of record, all comparative tests

Tables A.21-A.23 provides a summary of the best distribution/estimation pairing over all the tests performed. Again L-moment estimation methods predominate, although from a forecast point of view log-normal distributions are preferred.

Accuracy measures, 1933-1996 period of record, (alternate split sample test)

Tables A.24-A.26 provide results for the period of record 1933-1996 for the alternate split sample test. This portion of the record was considered so that the ranked values used in the comparisons related to the same record lengths; and consequently, the same accuracy in estimated plotting positions. Again, L-moment estimation seems to dominate, although log-normal distribution also is selected in a significant number of comparisons.

Summary results, 1933-1996 period of record, all comparative tests

Tables A.27-A.29 summarize all the comparisons with the 1933-1996 period of record. No outstanding difference with previous comparisons was realized.

Table A.15: Estimated bias for distribution/estimation pairings, 0.10 exceedance probability

distribution	[1]observed	[2]proportion	[3]ratio
l-gamma	138	.1273	1.27
l-GEV	139	.1282	1.28
l-g.logistic	150	.1384	1.38
l-g.pareto	119	.1098	1.10
l-gumbel	139	.1282	1.28
l-l.normal	118	.1089	1.09
l-wakeby	152	.1402	1.40
m-l.pIII	133	.1227	1.23
m-l.normal	105	.0969	.97
m-gumbel	139	.1282	1.28
c-l.normal	126	.1162	1.16
c-gumbel	119	.1098	1.10

[1]Observed exceedances of 0.10 discharge from 1084 total observation
[2]Observed exceedances divided by total observations
[3]Proportion/0.10

Table A.16: Estimated bias for distribution/estimation pairings, 0.020 exceedance probability

distribution	[1]observed	[2]proportion	[3]ratio
l-gamma	31	.0286	1.43
l-GEV	36	.0332	1.66
l-g.logistic	30	.0277	1.38
l-g.pareto	58	.0535	2.68
l-gumbel	23	.0212	1.06
l-l.normal	19	.0175	.88
l-wakeby	42	.0387	1.94
m-l.pIII	28	.0258	1.29
m-l.normal	15	.0138	.69
m-gumbel	25	.0231	1.15
c-l.normal	30	.0277	1.38
c-gumbel	28	.0258	1.29

[1]Observed exceedances of 0.02 discharge from 1084 total observation
[2]Observed exceedances divided by total observations
[3]Proportion/0.02

Table A.17: Estimated bias for distribution/estimation pairings, 0.010 exceedance probability

distribution	observed	proportion	ratio
l-gamma	20	.0185	1.85
l-GEV	29	.0268	2.68
l-g.logistic	19	.0175	1.75
l-g.pareto	53	.0489	4.89
l-gumbel	16	.0148	1.48
l-l.normal	14	.0129	1.29
l-wakeby	31	.0286	2.86
m-l.pIII	18	.0166	1.66
m-l.normal	12	.0111	1.11
m-gumbel	17	.0157	1.57
c-l.normal	18	.0166	1.66

[1]Observed exceedances of 0.01 discharge from 1084 total observation
[2]Observed exceedances divided by total observations
[3]Proportion/0.01

Table A.18: Accuracy comparison, alternate split sample test, full period of record at each gage top ranked event

Distribution/ Estimation method	[1]bias	[2]mse	[3]1-(PPOS/P)^2
l-gamma	.0130	.0019	2.8743
l-GEV	.0131	.0019	2.8282
l-g.logist	.0124	.0015	2.3745
l-g.pareto	.0095	.0027	3.6260
l-gumbel	.0150	.0018	2.8124
l-l.normal	.0224	.0024	4.1125
l-wakeby	.0145	.0026	3.6284
m-l.pIII	.0167	.0020	2.9771
m-l.normal	.0239	.0025	4.6985
m-gumbel	.0145	.0017	2.7579
c-l.normal	.0208	.0026	4.0422
c-gumbel	.0239	.0028	4.6719
selected	l-g.pareto	l-g.logist	l-g.logist

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.19: Accuracy comparison, alternate split sample test, full period of record at each gage10th ranked event

Distribution/ Estimation method	[1]bias	[2]mse	[3]$1\text{-}(PPOS/P)^2$
l-gamma	-.0206	.0089	.1838
l-GEV	-.0264	.0093	.1921
l-g.logist	-.0361	.0094	.1920
l-g.pareto	-.0064	.0105	.2302
l-gumbel	-.0250	.0091	.1872
l-l.normal	-.0233	.0069	.1437
l-wakeby	-.0322	.0107	.2306
m-l.pIII	-.0210	.0088	.1811
m-l.normal	-.0212	.0069	.1446
m-gumbel	-.0265	.0094	.1921
c-l.normal	-.0197	.0087	.1785
c-gumbel	-.0173	.0084	.1699
selected	l-g.pareto	m-l.normal	l-l.normal

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.20: Accuracy comparison, alternate split sample test, full period of record at each gage median event

Distribution/ Estimation method	[1]bias	[2]mse	[3]1-(PPOS/P)^2
l-gamma	-.0634	.0105	.0395
l-GEV	-.0621	.0113	.0430
l-g.logist	-.0610	.0126	.0478
l-g.pareto	-.0608	.0087	.0330
l-gumbel	-.0759	.0135	.0509
l-l.normal	-.0875	.0137	.0519
l-wakeby	-.0627	.0122	.0465
m-l.pIII	-.0642	.0109	.0416
m-l.normal	-.0863	.0134	.0507
m-gumbel	-.0752	.0132	.0499
c-l.normal	-.0526	.0130	.0494
c-gumbel	-.0593	.0156	.0593
selected	c-l.normal	l-g.pareto	l-g.pareto

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.21: Best distribution/estimation pairing selected based on bias, full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-l.normal	l-gumbel	l-GEV
forecast split record	m-l.normal	l-g.pareto	c-gumbel
hindcast split record	l-g.logist	l-wakeby	l-g.pareto
alternate odd split	l-g.pareto	l-g.pareto	c-l.normal
alternate even split	l-g.logist	l-g.logist	l-l.normal

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Table A.22: Best distribution/estimation pairing selected based on mean square error, full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-g.logist	l-wakeby	l-g.pareto
forecast split record	l-l.normal	m-l.normal	l-g.pareto
hindcast split record	l-g.logist	l-l.normal	l-g.pareto
alternate odd split	l-g.logist	m-l.normal	l-g.pareto
alternate even split	l-g.logist	m-l.normal	l-g.pareto

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Table A.23: Best distribution/estimation pairing selected based on average of (one - relative error squared), full period of record

Comparison	[1]Top ranked	[2]10th ranked	[3]Median
Period of record	l-g.logist	l-wakeby	l-g.pareto
forecast split record	m-l.normal	m-l.normal	l-g.pareto
hindcast split record	l-g.logist	l-l.normal	l-g.pareto
alternate odd split	l-g.logist	l-l.normal	l-g.pareto
alternate even split	l-g.logist	m-l.normal	l-g.pareto

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Table A.24: Accuracy comparison, alternate split sample test, 1933 to 1996 period of record, top ranked event

Distribution/ Estimation method	[1]bias	[2]mse	[3]1-(PPOS/P)^2
l-gamma	.0326	.0046	4.6877
l-GEV	.0319	.0042	4.2956
l-g.logist	.0277	.0034	3.4357
l-g.pareto	.0382	.0064	6.5065
l-gumbel	.0325	.0042	4.2975
l-l.normal	.0361	.0046	4.6355
l-wakeby	.0378	.0063	6.4622
m-l.pIII	.0383	.0047	4.8152
m-l.normal	.0340	.0042	4.2338
m-gumbel	.0314	.0039	4.0280
c-l.normal	.0456	.0062	6.3331
c-gumbel	.0487	.0067	6.8085
selection	l-g.logist	l-g.logist	l-g.logist

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.25: Accuracy comparison, alternate split sample test, 1933 to 1996 period of record, 10^{th} ranked event

Distribution/ Estimation method	[1]bias	[2]mse	[3]$1\text{-}(PPOS/P)^2$
l-gamma	.0899	.0139	.1415
l-GEV	.0794	.0107	.1075
l-g.logist	.0757	.0110	.1104
l-g.pareto	.0896	.0111	.1114
l-gumbel	.0784	.0135	.1375
l-l.normal	.0717	.0104	.1059
l-wakeby	.0863	.0116	.1170
m-l.pIII	.0762	.0099	.0992
m-l.normal	.0702	.0103	.1046
m-gumbel	.0751	.0130	.1323
c-l.normal	.0935	.0131	.1313
c-gumbel	.0927	.0139	.1382
selection	m-l.normal	m-l.pIII	m-l.pIII

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.26: Accuracy comparison, alternate split sample test, full period of record at each gage, median event

Distribution/ Estimation method	[1]bias	[2]mse	[3]$1\text{-}(PPOS/P)^2$
l-gamma	.0684	.0091	.0323
l-GEV	.0669	.0100	.0357
l-g.logist	.0805	.0127	.0451
l-g.pareto	.0407	.0057	.0202
l-gumbel	.0620	.0092	.0327
l-l.normal	.0536	.0083	.0294
l-wakeby	.0591	.0094	.0336
m-l.pIII	.0595	.0083	.0296
m-l.normal	.0545	.0084	.0299
m-gumbel	.0638	.0089	.0317
c-l.normal	.0848	.0190	.0675
c-gumbel	.0785	.0278	.0990
selection	l-g.pareto	l-g.pareto	l-g.pareto

[1]average difference between distribution estimate and plotting position in reserved data

[2]average sum of squared differences between distribution estimate and plotting position in reserved data

[3]average sum of 1 - (plotting position of reserved data/distribution prediction)^2

Table A.27: Best distribution/estimation pairing selected based on bias, 1933-1996 period of record

Comparison	Top ranked	10th ranked	Median
Period of record	m-l.normal	m-gumbel	l-g.logist
forecast split record	c-gumbel	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	m-l.normal	l-g.pareto
alternate odd split	l-g.logist	m-l.normal	l-g.pareto
alternate even split	m-l.normal	l-g.pareto	l-wakeby

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Table A.28: Best distribution/estimation pairing selected based on mean square error, 1933-1996 period of record

Comparison	Top ranked	10th ranked	Median
Period of record	l-g.logist	l-wakeby	l-wakeby
forecast split record	m-l.normal	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	l-g.pareto	l-g.pareto
alternate odd split	l-g.logist	m-l.pIII	l-g.pareto
alternate even split	l-wakeby	m-l.normal	l-g.pareto

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Table A.29: Best distribution/estimation pairing selected based on average (one - relative error squared), full period of record

Comparison	Top ranked	10th ranked	Median
Period of record	l-g.logist	l-wakeby	l-wakeby
forecast split record	m-l.normal	l-g.pareto	l-g.pareto
hindcast split record	l-g.logist	l-g.pareto	l-g.pareto
alternate odd split	l-g.logist	m-l.pIII	l-g.pareto
alternate even split	l-wakeby	m-l.normal	l-g.pareto

[1]Comparison with top ranked event in reserved period of record
[2]Comparison with 10th ranked event in reserved period of record
[3]Comparison with median ranked event in reserved period of record

Appendix B. Comparisons with empirical distributions

Comparison of predictions with empirical distributions obtained with different distribution/estimation method pairings for selected gages are presented in this appendix. The following abbreviations are used in the figures to signify the combined distribution/estimation pairs:

l-gamma	Gamma distribution with L-moments;
l-GEV	Generalized Extreme Value distribution with L-moments;
l-g.logist	Generalized Logistic distribution with L-moments;
l-g.pareto	Generalize Pareto distribution with L-moments;
l-gumbel	Gumbel distribution with L-moments;
l-l.normal	log-Normal distribution with L-moments;
l-wakeby	Wakeby distribution with L-moments;
m-lpIII	log-Pearson III with Bulletin 17B procedure;
m-l.normal	log-Normal distribution with standard moments;
m-gumbel	Gumbel distribution with standard moments;
c-l.normal	log-Normal with regression applied to data censored at 50% chance exceedance;
c-gumbel	Gumbel with regression applied to data censored at 50% chance exceedance.

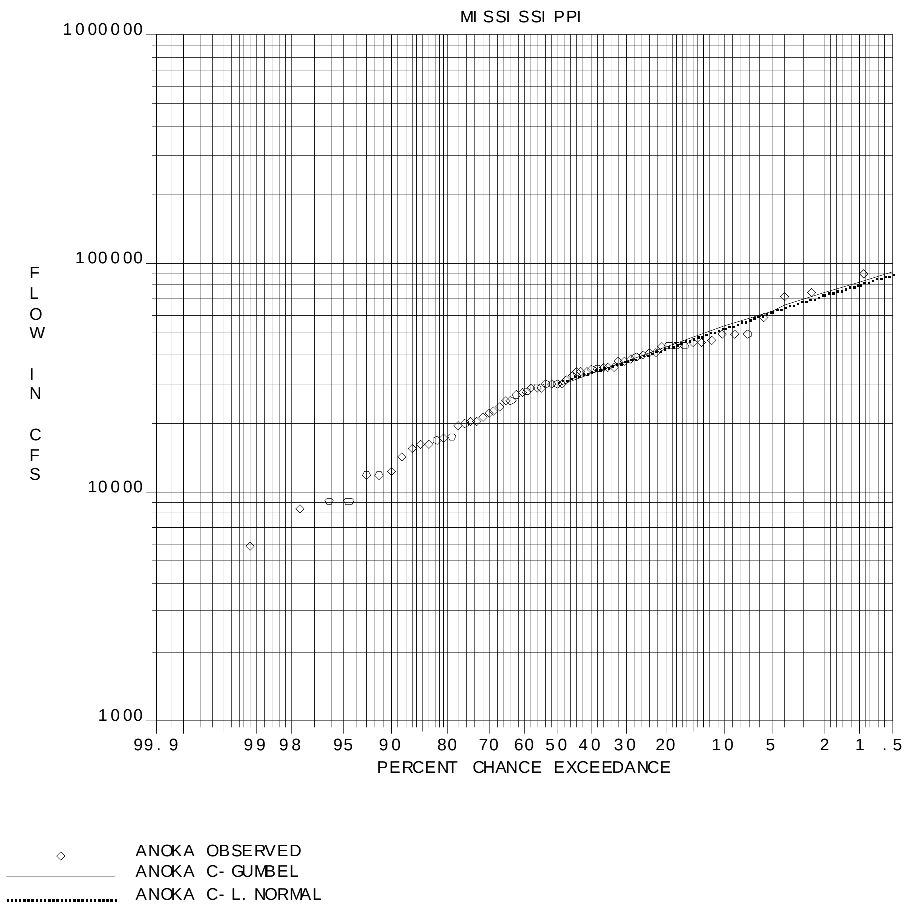

Figure B.1: Comparison of Cunnane Plotting Position with Gumbel and normal distributions, regression estimates for censored data, Anoka, Minnesota, Mississipi River

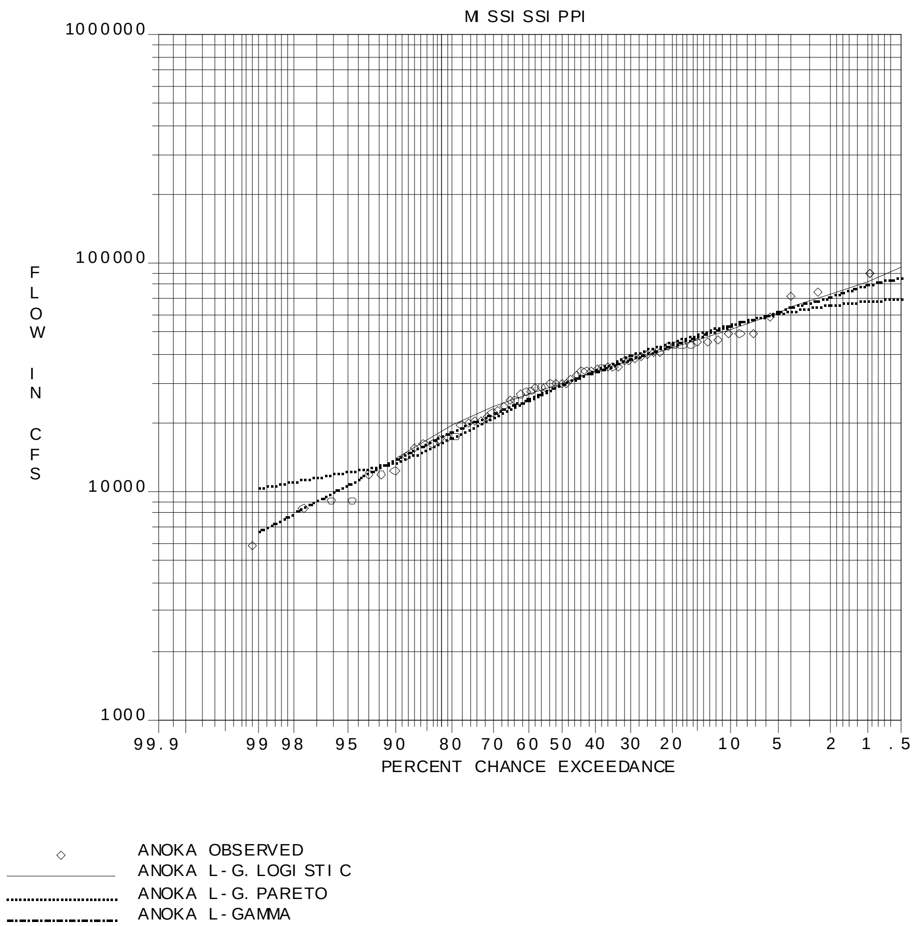

Figure B.2: Comparison of Cunnane Plotting Position with generalized logistic, Pareto and gamma distributions, L-moment estimation, Anoka, Minnesota, Mississipi River

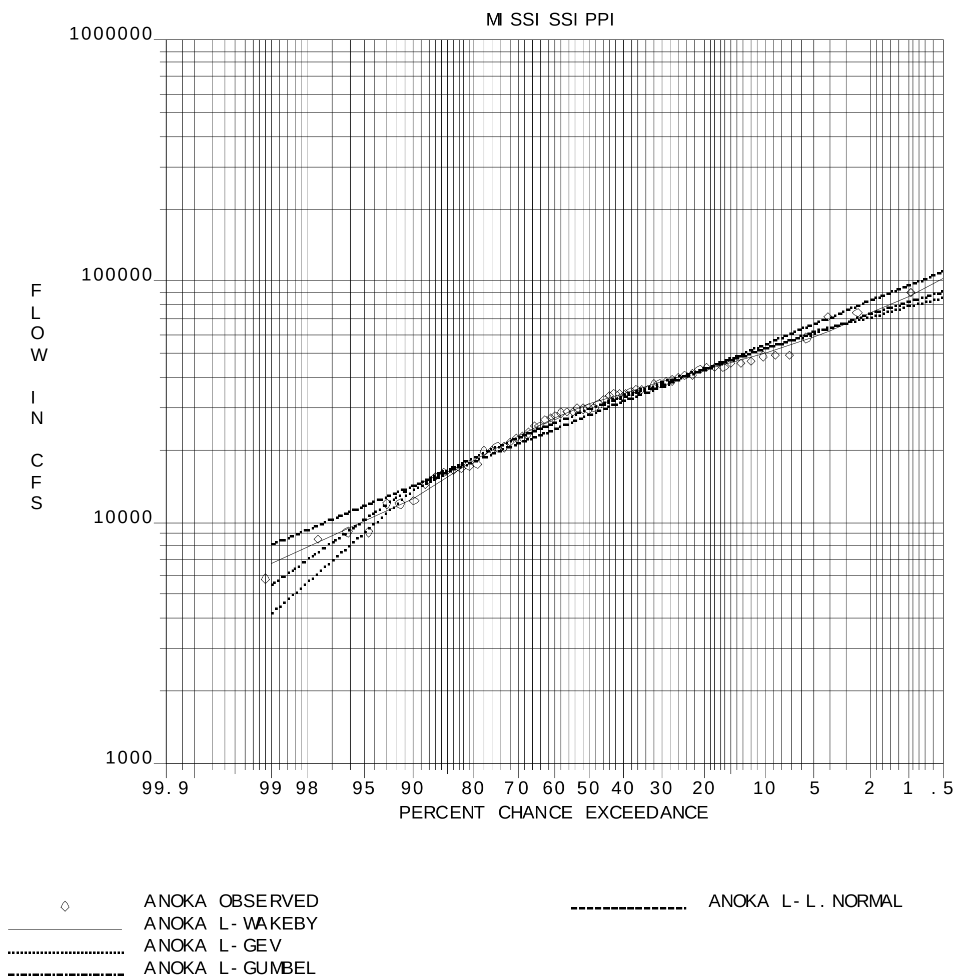

Figure B.3: Comparison with Cunnane Plotting Positions Wakeby, generalized extreme value, Gumbel, log-Normal, with L-moment estimation, Anoka, Minnesota, Mississipi River

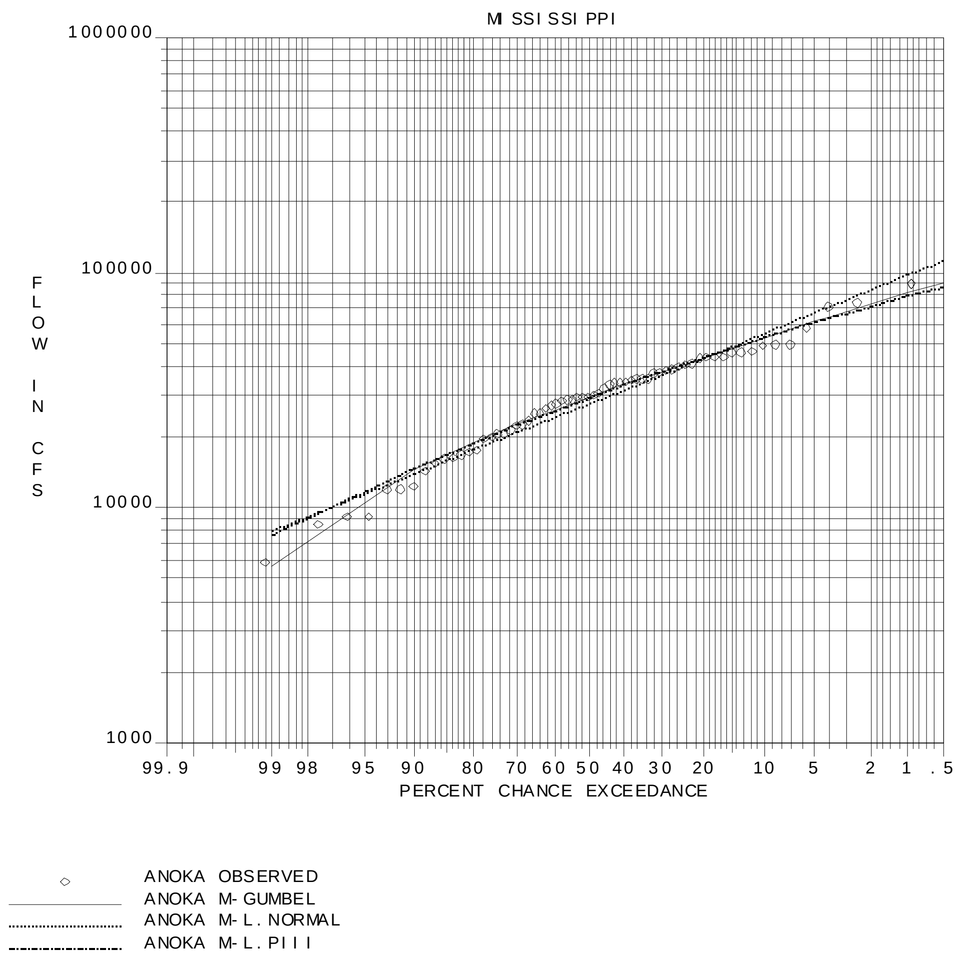

Figure B.4: Comparison with Cunnane Plotting Positions, Gumbel and log-normal distributions, standard moment estimation, log-Pearson III, Bulletin 17B estimation, Anoka, Minnesota, Mississippi River

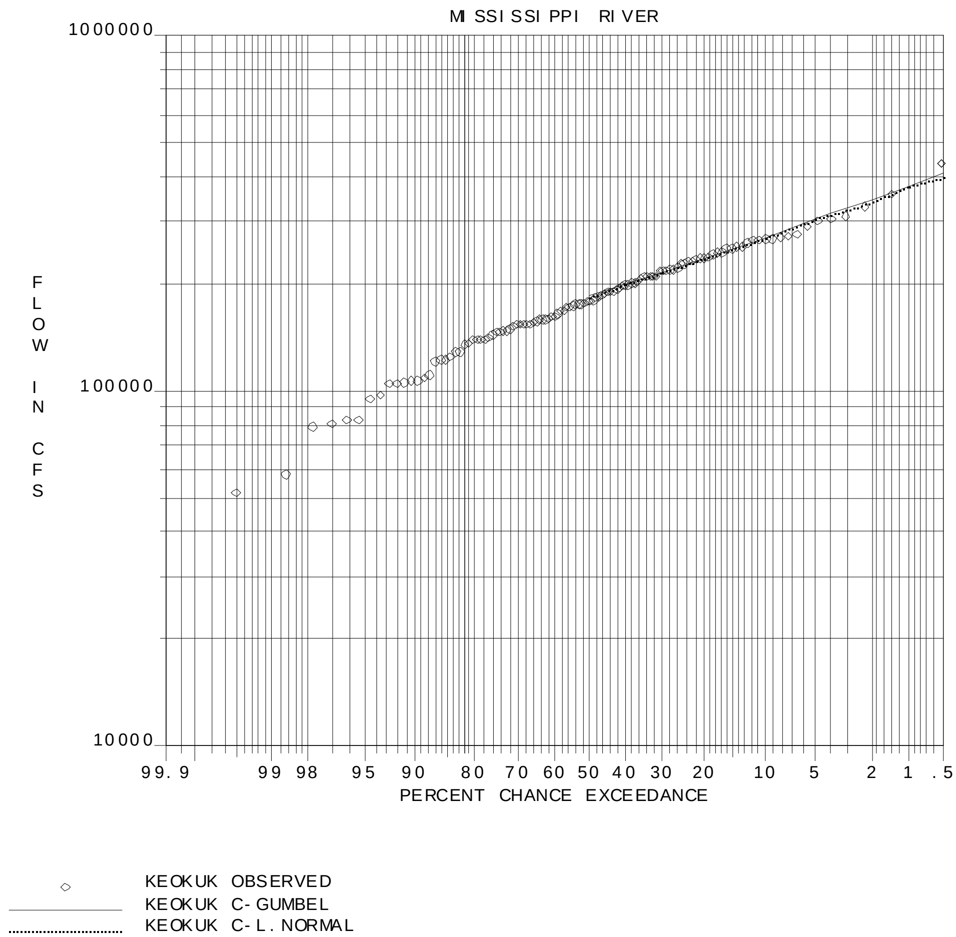

Figure B.5: Comparison of Cunnane Plotting Position with Gumbel and normal distributions, regression estimates for censored data, Keokuk, Iowa, Mississippi River

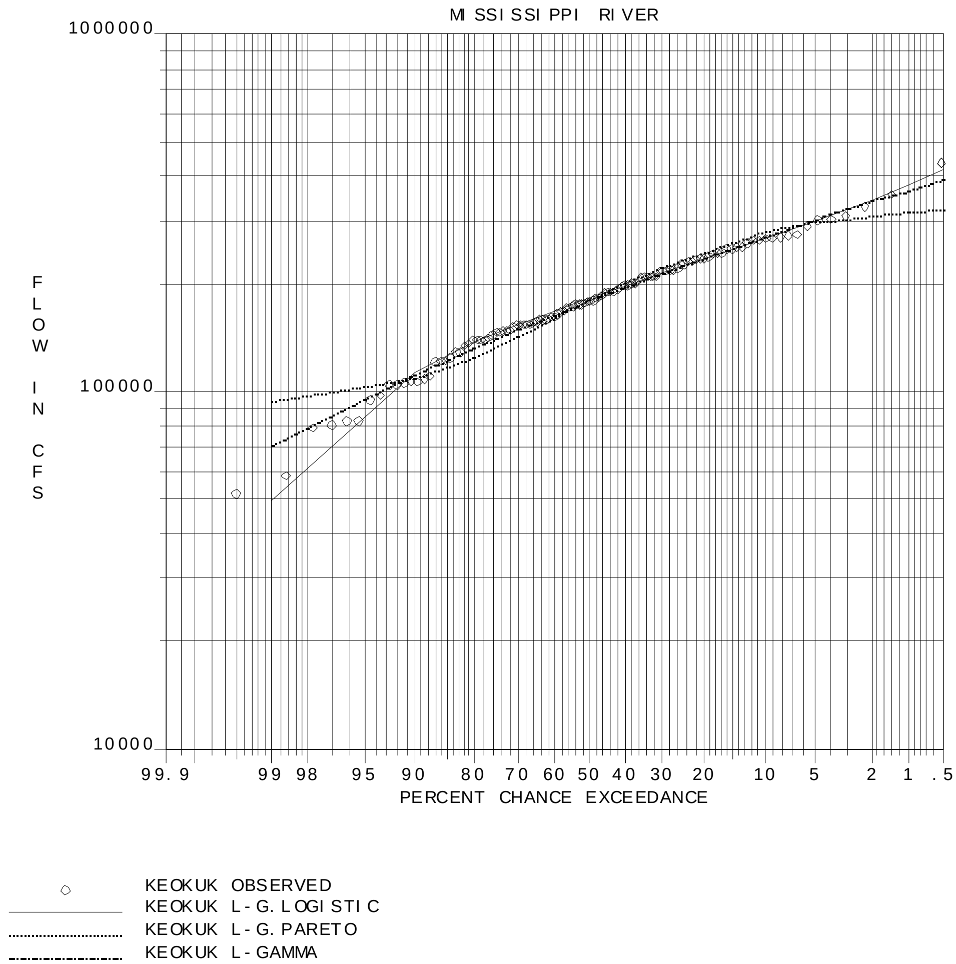

Figure B.6: Comparison of Cunnane Plotting Position with, generalized logistic, Pareto and gamma distributions, L-moment estimation, Keokuk, Iowa, Mississippi River

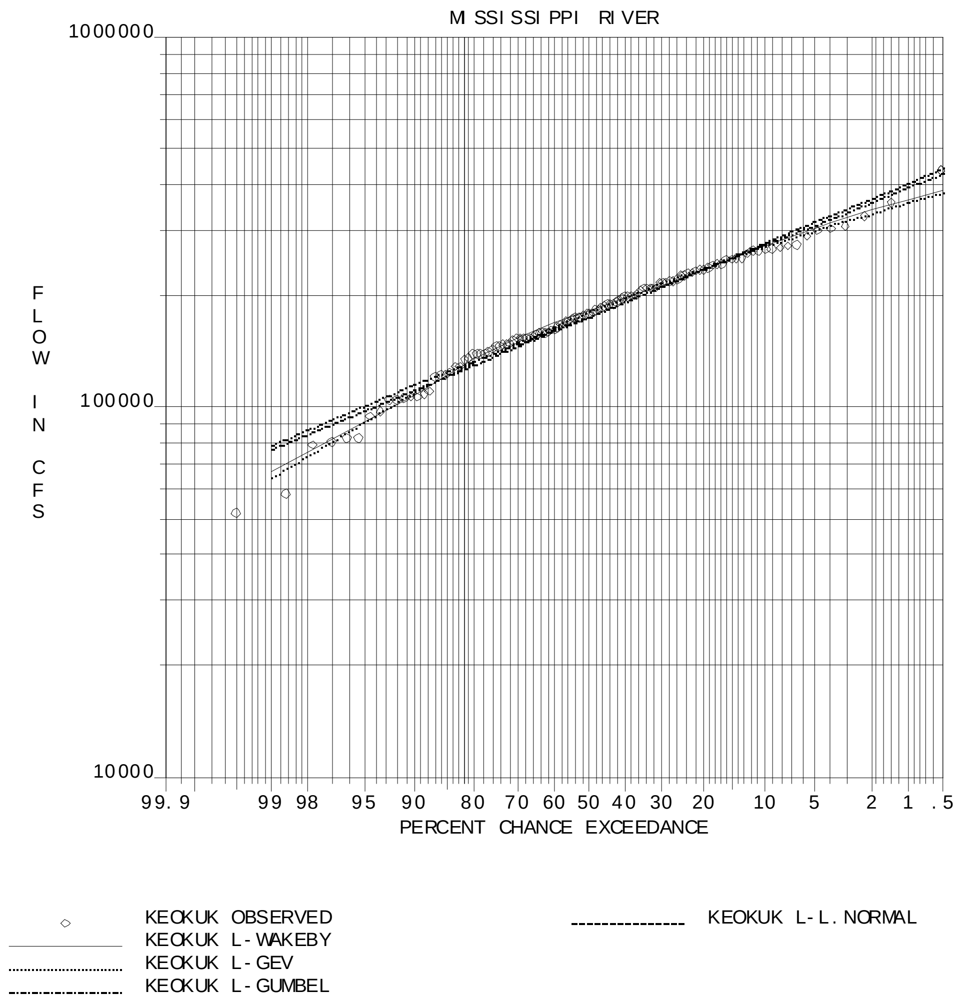

Figure B.7: Comparison with Cunnane Plotting Positions Wakeby, generalized extreme value, Gumbel, log-Normal, with L-moment estimation, Keokuk, Iowa, Mississippi River

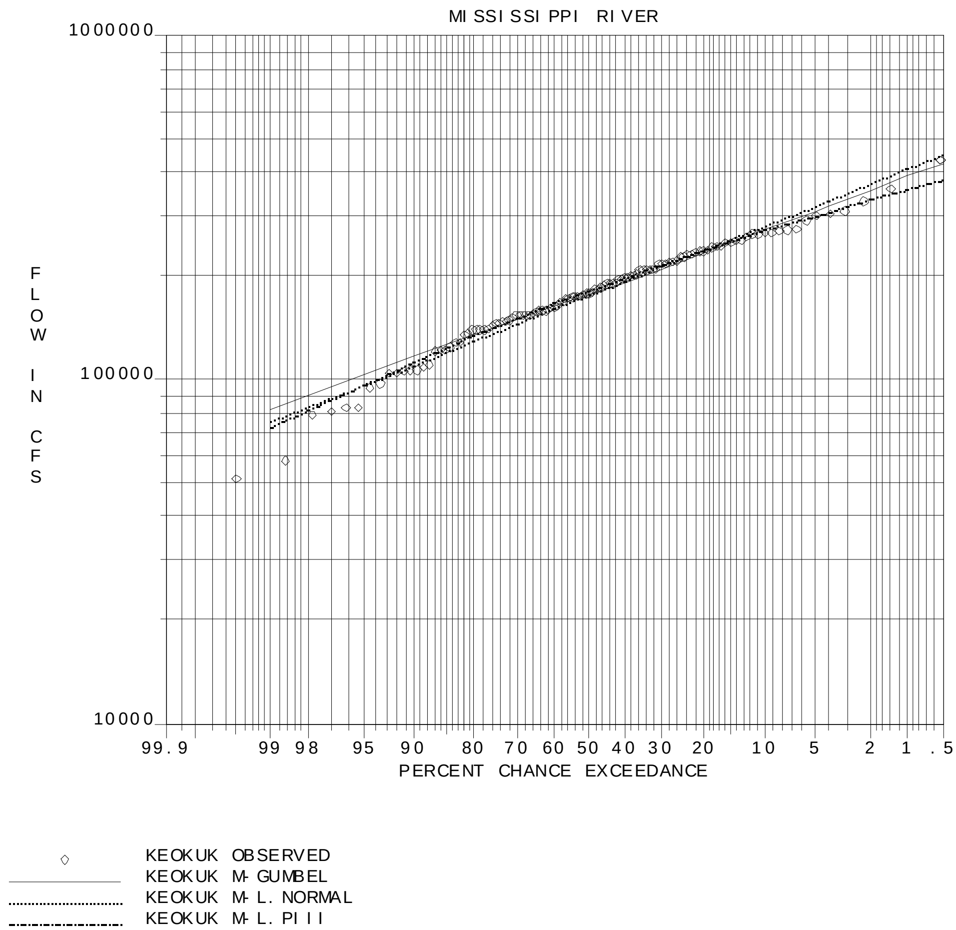

Figure B.8: Comparison with Cunnane Plotting Positions, Gumbel and log-normal distributions, standard moment estimation, log-Pearson III, Bulletin 17B estimation, Keokuk, Iowa, Mississippi River

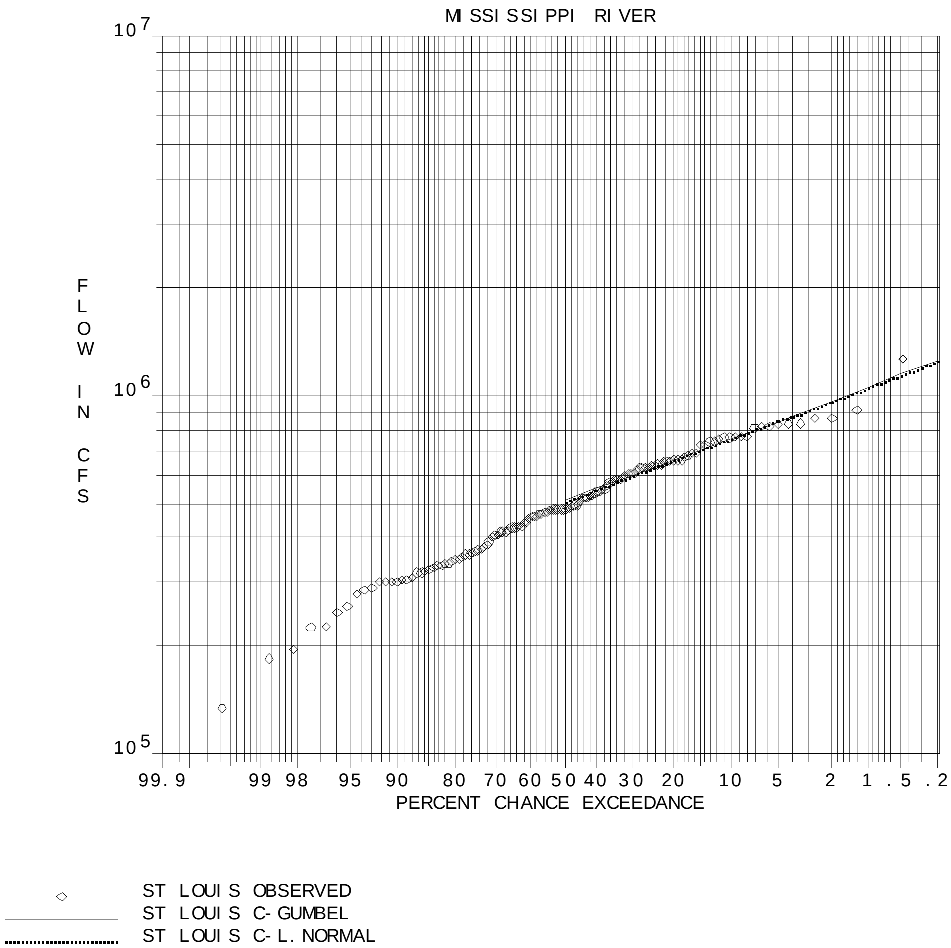

Figure B.9: Comparison of Cunnane Plotting Position with Gumbel and normal distribution, regression estimates for censored data, St. Louis, Missouri, Mississippi River

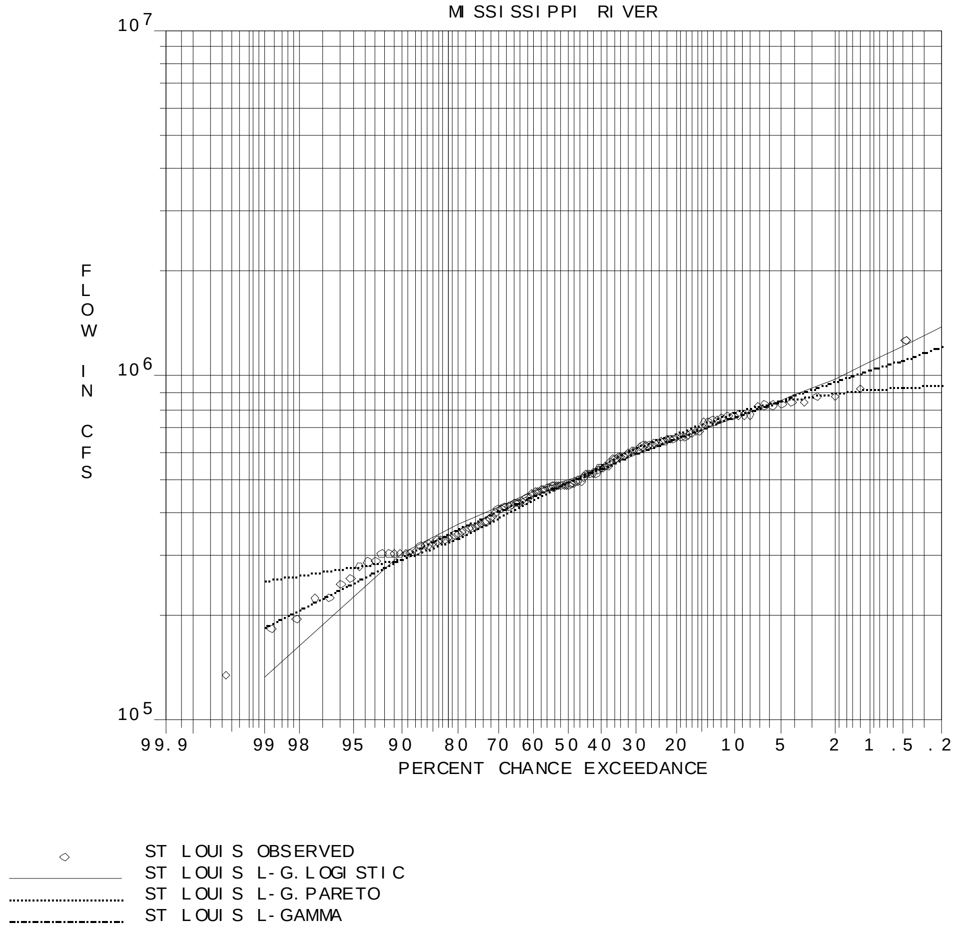

Figure B.10: Comparison of Cunnane Plotting Position with Generalized logistic, Pareto and gamma Distributions, L-moment estimation St. Louis, Missouri, Mississippi River

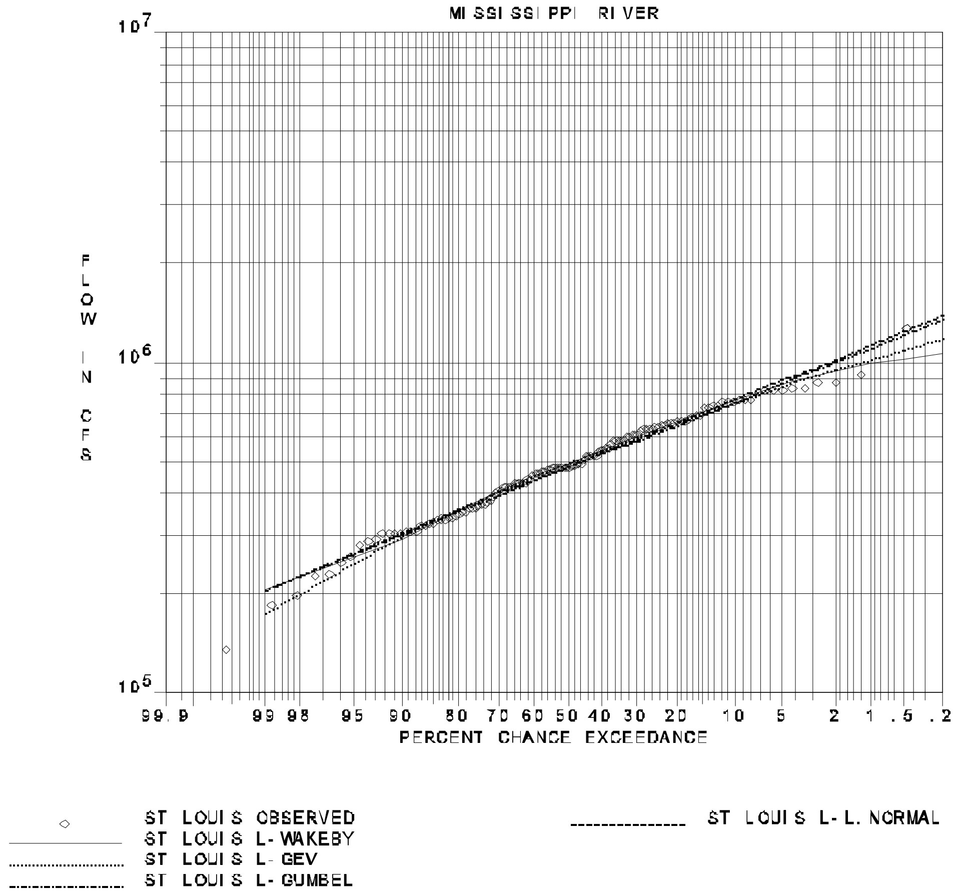

Figure B.11: Comparison with Cunnane Plotting Positions, Wakeby, generalized extreme value, Gumbel, log-normal, with L-moment estimation, St. Louis, Missouri, Mississippi River

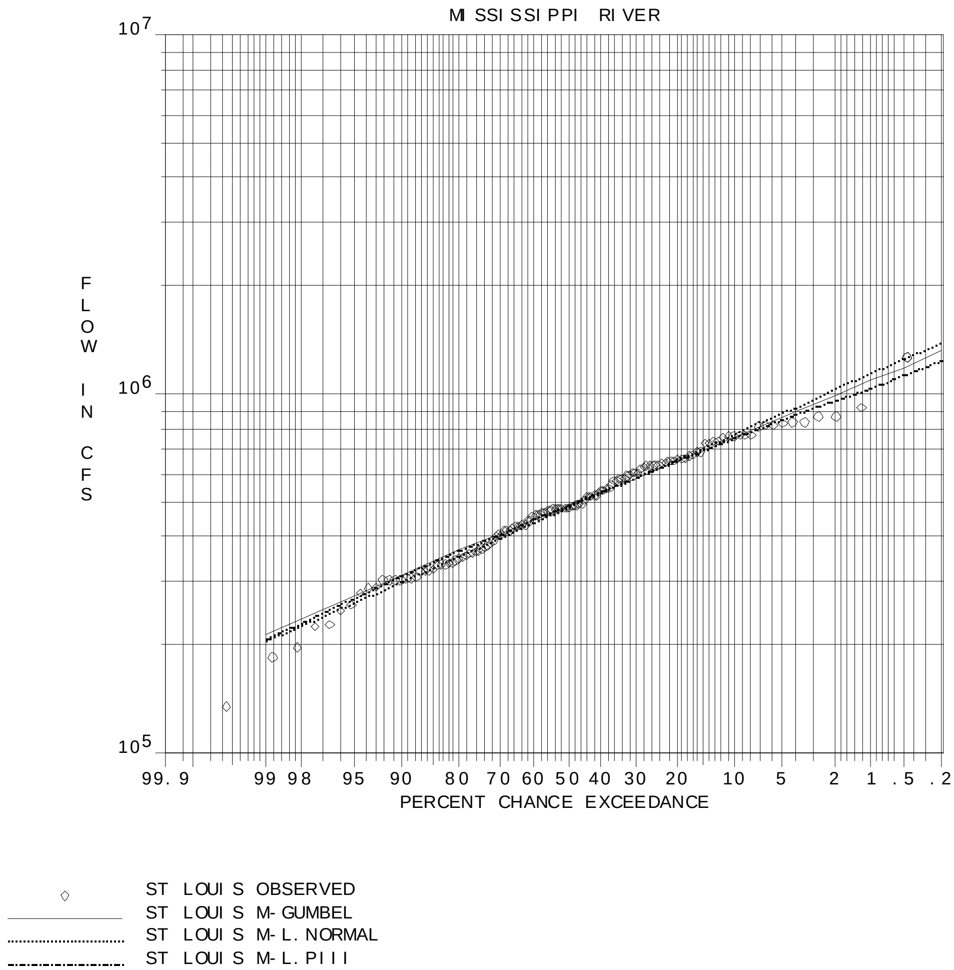

Figure B.12: Comparison with Cunnane Plotting Positions, Gumbel and log-normal distributions, standard moment estimation, log-Pearson III, Bulletin 17B estimation, St. Louis, Missouri, Mississippi River

Appendix C: Comparative plots for sensitivity analysis

Comparative plot of distribution/estimation methods recommended by the Technical Advisory Group at selected gages (see section 5 for further description). The following plots at selected gages provide comparisons of the following distribution estimation pairings:

C.1 LP III

The log-Pearson III distribution estimated by standard moments, as per recommended in the Bulletin 17B guidelines, including low-outlier censoring (IACWD, 1982). The adopted skew (or weighted skew) was assumed to be equal to the station skew.

C.2 LP III REGIONAL G

The log-Pearson III distribution estimated using standard moments as in C.1, except that a regional average skew coefficient was substituted for the station skew.

C.3 LP III EMA REGIONAL G

The log-Pearson III distribution is estimated from data censored to include the largest 50% of the observed flows (a top half fit) using the expected moments algorithm.

C.4 GNORMAL REGIONAL K

The generalized normal distribution estimated using L-moments, and substituting a regional average shape parameter for the at-station value.

C.5 GEV REGIONAL K

The generalized extreme value distribution estimated using L-moments, and substituting a regional average shape parameter for the at-station value.

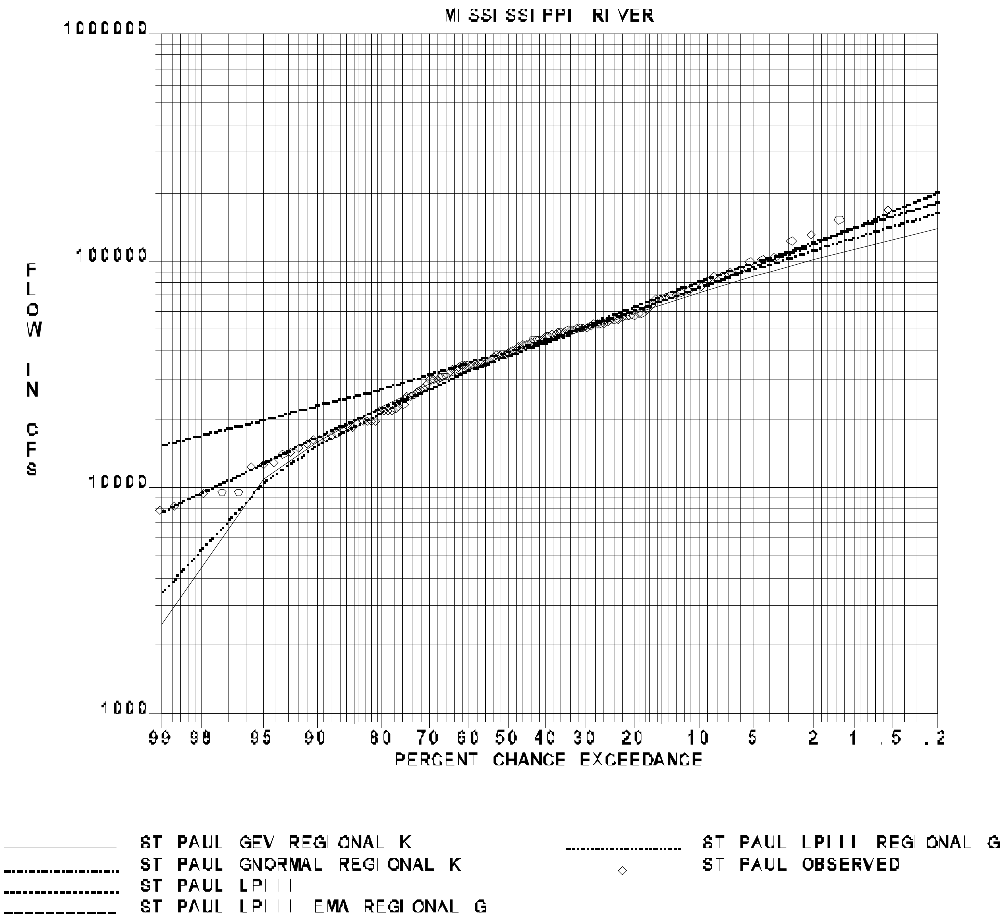

Figure C.1: Sensitivity analysis comparison at St. Paul, Minnesota, Mississippi River

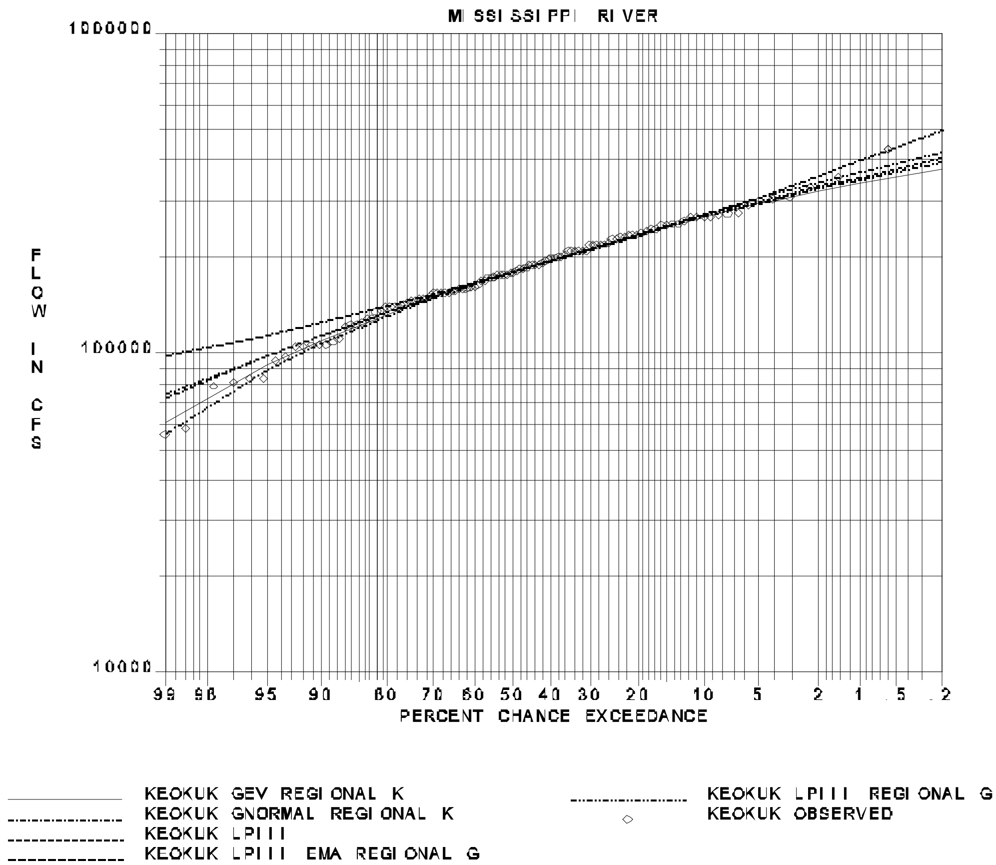

Figure C.2: Sensitivity analysis comparison at Keokuk, Iowa, Mississippi River

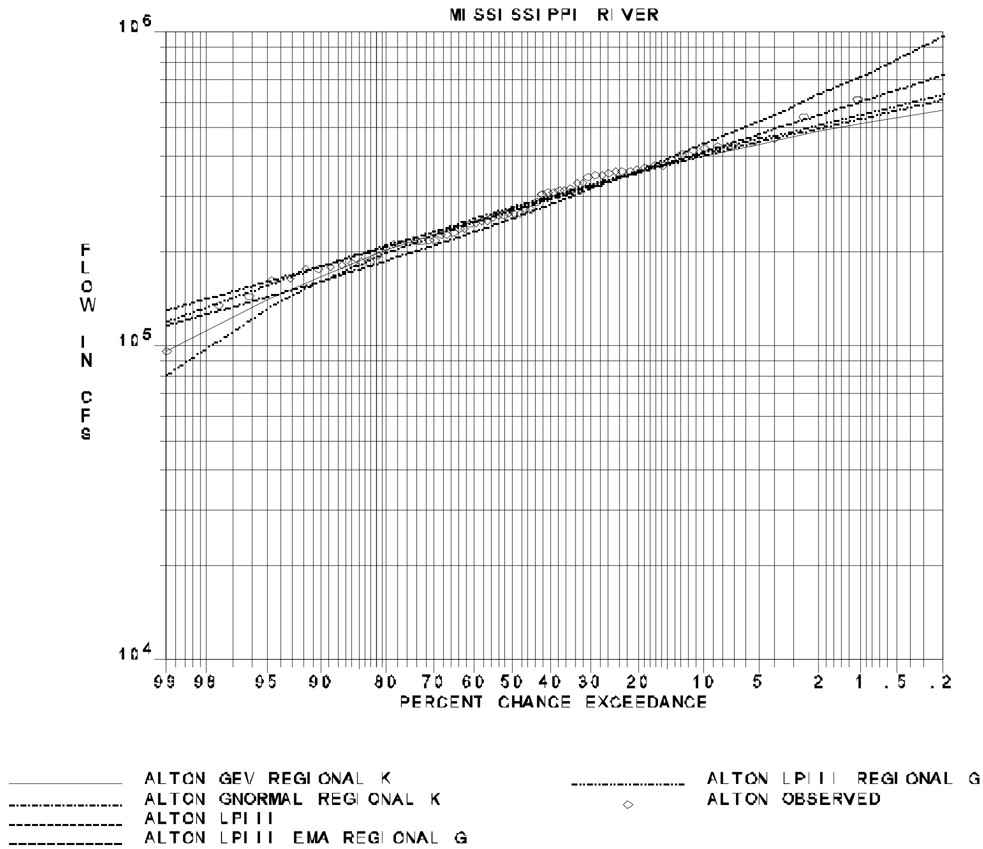

Figure C.3: Sensitivity analysis comparison at Alton/Grafton, Illinois, Mississippi River

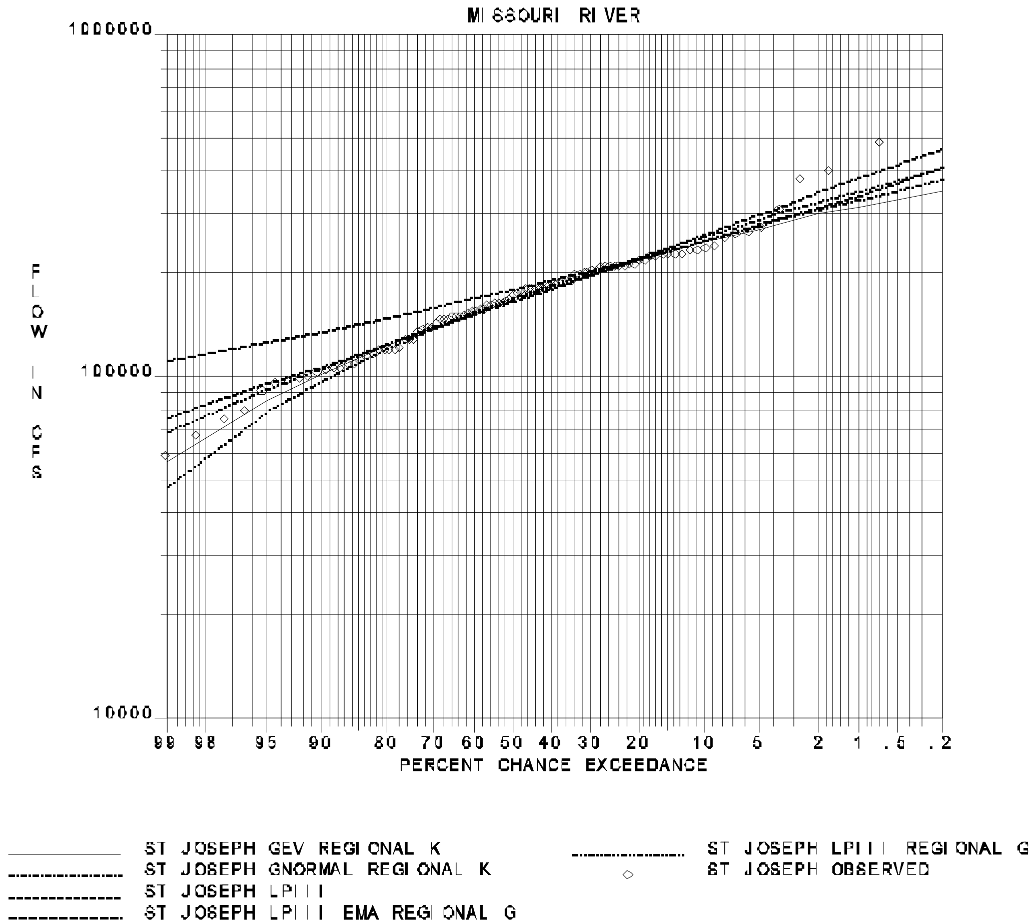

Figure C.4: Sensitivity analysis comparison at St. Joseph, Missouri, Missouri River

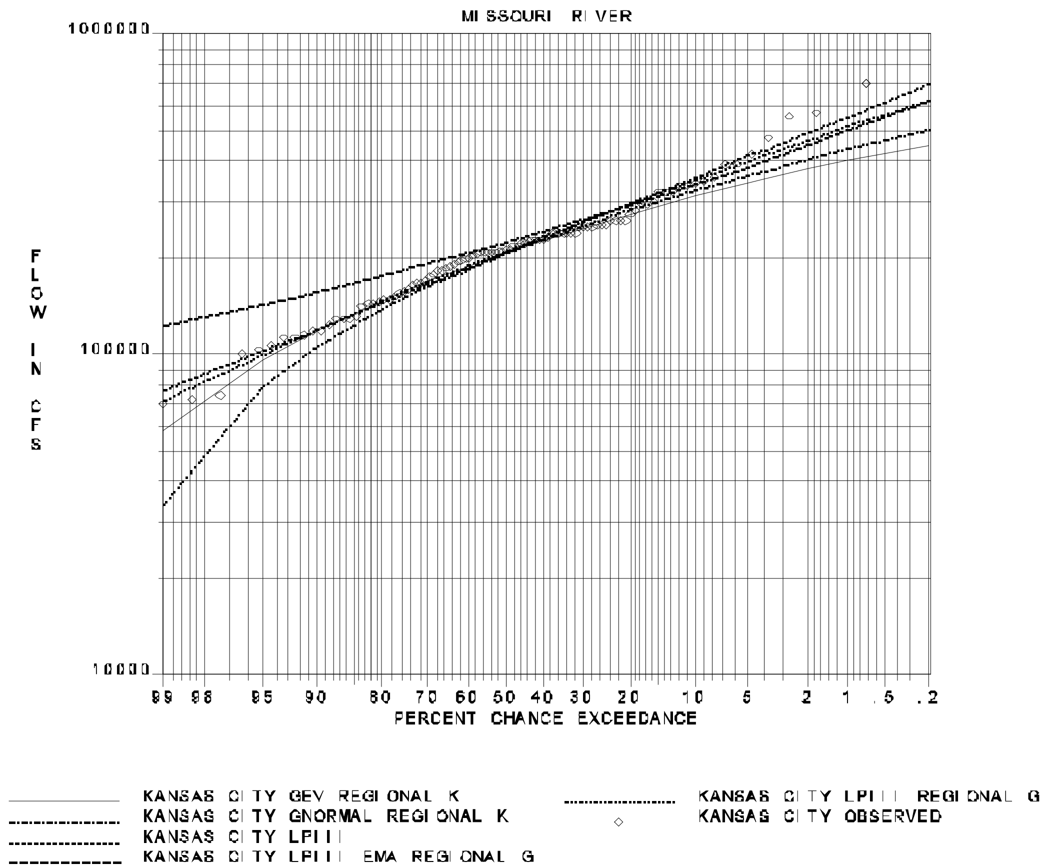

Figure C.5: Sensitivity analysis comparison at Kansas City, Missouri, Missouri River

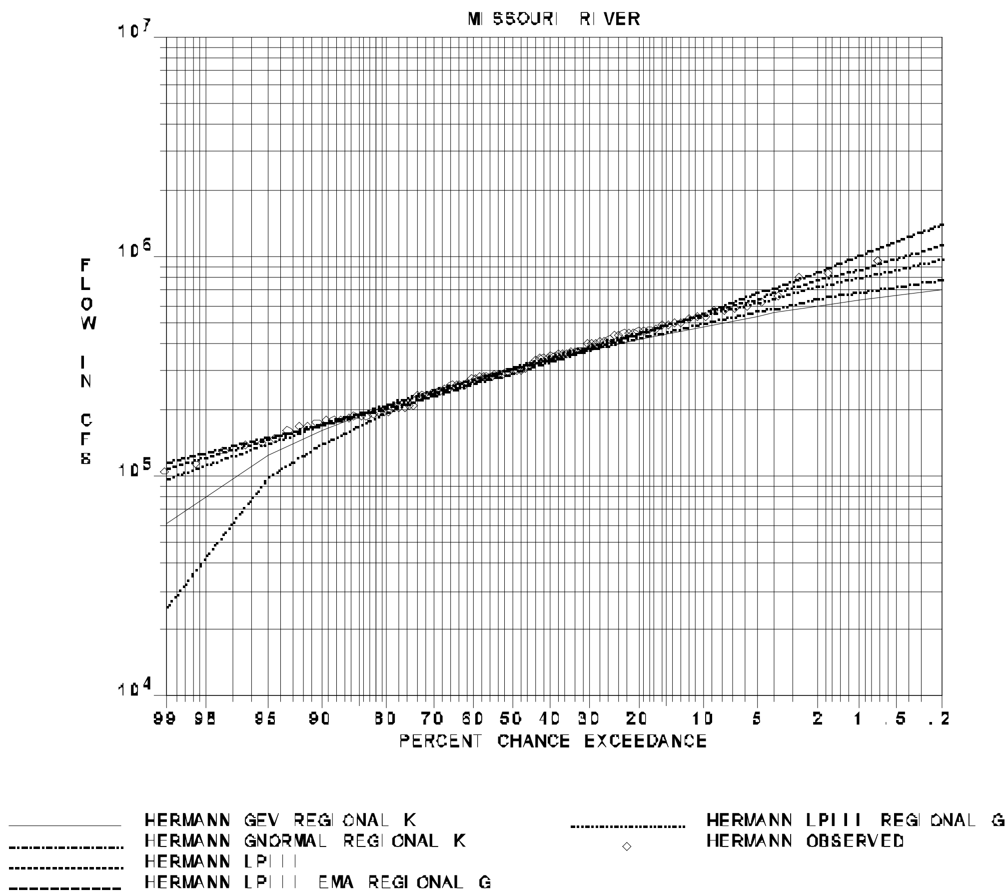

Figure C.6: Sensitivity analysis comparison at Hermann, Missouri, Missouri River

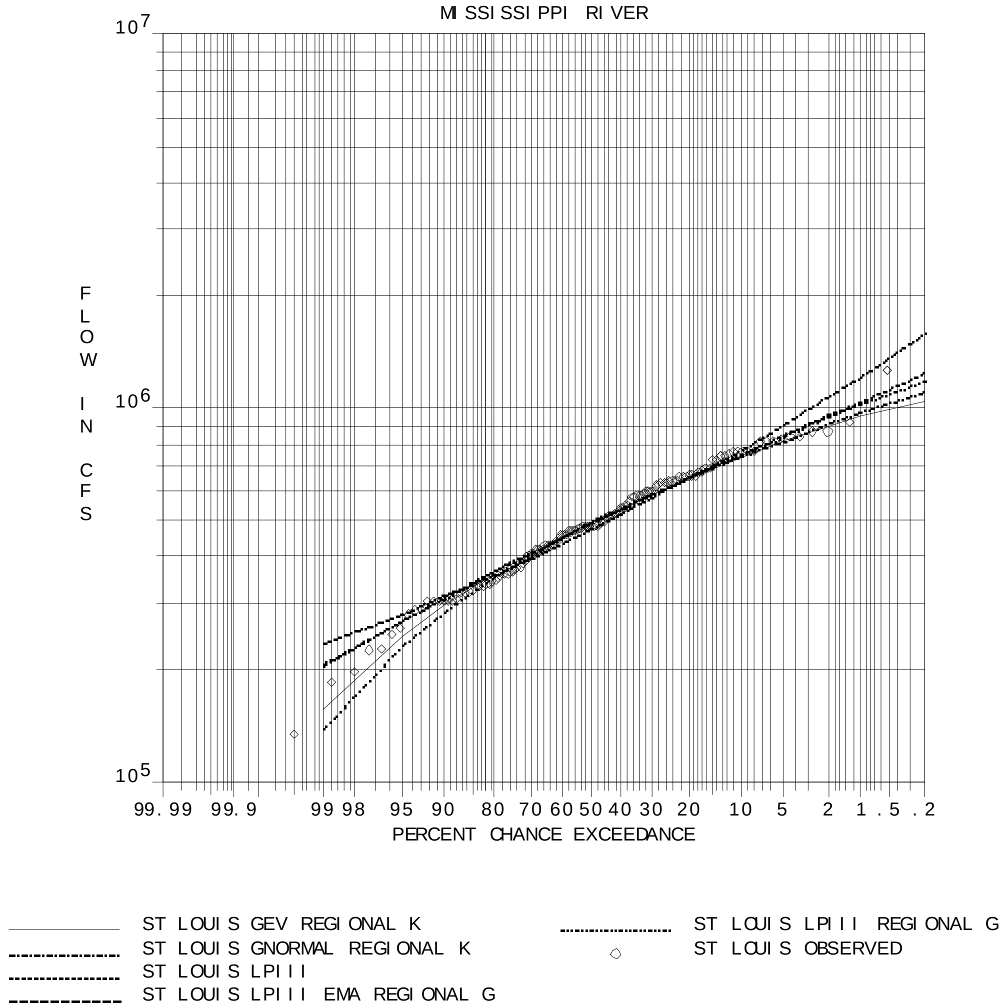

Figure C.7: Sensitivity analysis comparison at St. Louis, Missouri, Mississippi River

Appendix D: Tables of inter-station correlation and computed error covariance matrices for regional skew and generalizes normal and generalized extreme value shape factors

The generalized least squares regression analysis described in sections 5 and 6 required an estimate of the covariance of the time sampling errors associated with the regression residuals. The stations used in computing the error covariances are shown in Table D.1. Table D.2 displays the sample inter-station correlation values needed for estimating the covariance matrix. The algorithm described in section 6.4.1 used these correlations to compute the error covariance matrices shown in Tables D.3-D.5.

Table D.1: Stations, drainage area and shape parameters used in least squares analysis (period of record 1933-1996)

Station	Name	Drainage Area (square miles)	skew (1)	gnormal (2)	gev (3)
1	Sioux City	314580	-0.13	-0.25	0.08
2	Omaha	322800	-0.13	-0.20	0.12
3	Nebraska City	410000	-0.19	-0.19	0.13
4	St. Joseph	420300	-0.10	-0.19	0.13
5	Kansas City	485200	0.10	-0.35	0.00
6	Booneville	501200	-0.02	-0.29	0.04
7	Hermann	524200	0.04	-0.34	0.01
8	Anoka	19600	-0.25	-0.16	0.15
9	St. Paul	36800	0.20	-0.46	-0.08
10	Mankato	14900	-0.07	-0.65	-0.21
11	Winona	59200	0.09	-0.30	0.04
12	Dubuque	67500	-0.18	-0.07	0.22
13	Clinton	10400	-0.35	-0.04	0.25
14	Keokuk	82000	-0.31	-0.04	0.25
15	Hannibal	86000	-0.19	-0.01	0.27
16	Louisiana	119000	-0.11	-0.10	0.19
17	Meredosia	137000	-0.50	0.06	0.34
18	Alton	141000	0.12	-0.10	0.20
19	St. Louis	26030	-0.18	-0.05	0.24
20	Chester	171300	-0.30	-0.03	0.26

(1) log-Pearson III skew coefficient
(2) Generalized Normal shape parameter
(3) Generalized Extreme Value shape parameter

Table D.2: Cross correlations between annual maximum daily flow values

station	year	year	station	year	year	correlation
Sioux City	1933	1995	Omaha	1933	1995	.97
Sioux City	1933	1995	Nebraska City	1933	1995	.93
Sioux City	1933	1995	St Joseph	1933	1995	.82
Sioux City	1933	1995	Kansas City	1933	1995	.57
Sioux City	1933	1995	Booneville	1933	1995	.46
Sioux City	1933	1995	Hermann	1933	1995	.40
Sioux City	1933	1995	Anoka	1933	1995	.56
Sioux City	1933	1995	St Paul	1933	1995	.52
Sioux City	1933	1995	Mankato	1933	1995	.40
Sioux City	1933	1995	Winona	1933	1995	.36
Sioux City	1933	1995	Dubuque	1933	1995	.43
Sioux City	1933	1995	Clinton	1933	1995	.46
Sioux City	1933	1995	Keokuk	1933	1995	.40
Sioux City	1933	1995	Hannibal	1933	1995	.41
Sioux City	1933	1995	Louisiana	1933	1995	.32
Sioux City	1933	1995	Meredosia	1933	1995	.30
Sioux City	1933	1995	Alton	1933	1995	.33
Sioux City	1933	1995	St Louis	1933	1995	.41
Sioux City	1933	1995	Chester	1933	1995	.40

station	year	year	station	year	year	correlation
Omaha	1933	1995	Nebraska City	1933	1995	.96
Omaha	1933	1995	St Joseph	1933	1995	.86
Omaha	1933	1995	Kansas City	1933	1995	.57
Omaha	1933	1995	Booneville	1933	1995	.47
Omaha	1933	1995	Hermann	1933	1995	.40
Omaha	1933	1995	Anoka	1933	1995	.54
Omaha	1933	1995	St Paul	1933	1995	.54
Omaha	1933	1995	Mankato	1933	1995	.44
Omaha	1933	1995	Winona	1933	1995	.40
Omaha	1933	1995	Dubuque	1933	1995	.45
Omaha	1933	1995	Clinton	1933	1995	.49
Omaha	1933	1995	Keokuk	1933	1995	.44
Omaha	1933	1995	Hannibal	1933	1995	.43
Omaha	1933	1995	Louisiana	1933	1995	.33
Omaha	1933	1995	Meredosia	1933	1995	.29
Omaha	1933	1995	Alton	1933	1995	.34
Omaha	1933	1995	St Louis	1933	1995	.41
Omaha	1933	1995	Chester	1933	1995	.39

station	year	year	station	year	year	correlation
Nebraska City	1933	1995	St Joseph	1933	1995	.91
Nebraska City	1933	1995	Kansas City	1933	1995	.63
Nebraska City	1933	1995	Booneville	1933	1995	.56
Nebraska City	1933	1995	Hermann	1933	1995	.50

Nebraska City	1933	1995	Anoka	1933	1995	.55
Nebraska City	1933	1995	St Paul	1933	1995	.59
Nebraska City	1933	1995	Mankato	1933	1995	.52
Nebraska City	1933	1995	Winona	1933	1995	.42
Nebraska City	1933	1995	Dubuque	1933	1995	.50
Nebraska City	1933	1995	Clinton	1933	1995	.52
Nebraska City	1933	1995	Keokuk	1933	1995	.53
Nebraska City	1933	1995	Hannibal	1933	1995	.53
Nebraska City	1933	1995	Louisiana	1933	1995	.41
Nebraska City	1933	1995	Meredosia	1933	1995	.37
Nebraska City	1933	1995	Alton	1933	1995	.43
Nebraska City	1933	1995	St Louis	1933	1995	.51
Nebraska City	1933	1995	Chester	1933	1995	.50

Table D.2: Cross correlations between annual maximum daily flow values (continued)

station	year	year	station	year	year	correlation
St Joseph	1933	1995	Kansas City	1933	1995	.79
St Joseph	1933	1995	Booneville	1933	1995	.72
St Joseph	1933	1995	Hermann	1933	1995	.62
St Joseph	1933	1995	Anoka	1933	1995	.55
St Joseph	1933	1995	St Paul	1933	1995	.66
St Joseph	1933	1995	Mankato	1933	1995	.62
St Joseph	1933	1995	Winona	1933	1995	.47
St Joseph	1933	1995	Dubuque	1933	1995	.54
St Joseph	1933	1995	Clinton	1933	1995	.57
St Joseph	1933	1995	Keokuk	1933	1995	.60
St Joseph	1933	1995	Hannibal	1933	1995	.60
St Joseph	1933	1995	Louisiana	1933	1995	.50
St Joseph	1933	1995	Meredosia	1933	1995	.36
St Joseph	1933	1995	Alton	1933	1995	.48
St Joseph	1933	1995	St Louis	1933	1995	.60
St Joseph	1933	1995	Chester	1933	1995	.55

station	year	year	station	year	year	correlation
Kansas City	1933	1995	Booneville	1933	1995	.89
Kansas City	1933	1995	Hermann	1933	1995	.80
Kansas City	1933	1995	Anoka	1933	1995	.42
Kansas City	1933	1995	St Paul	1933	1995	.58
Kansas City	1933	1995	Mankato	1933	1995	.58
Kansas City	1933	1995	Winona	1933	1995	.46
Kansas City	1933	1995	Dubuque	1933	1995	.54
Kansas City	1933	1995	Clinton	1933	1995	.57
Kansas City	1933	1995	Keokuk	1933	1995	.60
Kansas City	1933	1995	Hannibal	1933	1995	.61
Kansas City	1933	1995	Louisiana	1933	1995	.58
Kansas City	1933	1995	Meredosia	1933	1995	.43

Kansas City	1933	1995	Alton	1933	1995	.52
Kansas City	1933	1995	St Louis	1933	1995	.71
Kansas City	1933	1995	Chester	1933	1995	.66

station	year	year	station	year	year	correlation
Booneville	1933	1995	Hermann	1933	1995	.92
Booneville	1933	1995	Anoka	1933	1995	.36
Booneville	1933	1995	St Paul	1933	1995	.54
Booneville	1933	1995	Mankato	1933	1995	.55
Booneville	1933	1995	Winona	1933	1995	.44
Booneville	1933	1995	Dubuque	1933	1995	.50
Booneville	1933	1995	Clinton	1933	1995	.55
Booneville	1933	1995	Keokuk	1933	1995	.64
Booneville	1933	1995	Hannibal	1933	1995	.71
Booneville	1933	1995	Louisiana	1933	1995	.70
Booneville	1933	1995	Meredosia	1933	1995	.57
Booneville	1933	1995	Alton	1933	1995	.66
Booneville	1933	1995	St Louis	1933	1995	.85
Booneville	1933	1995	Chester	1933	1995	.81

Table D.2: Cross correlations between annual maximum daily flow values (continued)

station	year	year	station	year	year	correlation
Hermann	1933	1995	Anoka	1933	1995	.30
Hermann	1933	1995	St Paul	1933	1995	.46
Hermann	1933	1995	Mankato	1933	1995	.47
Hermann	1933	1995	Winona	1933	1995	.35
Hermann	1933	1995	Dubuque	1933	1995	.47
Hermann	1933	1995	Clinton	1933	1995	.51
Hermann	1933	1995	Keokuk	1933	1995	.64
Hermann	1933	1995	Hannibal	1933	1995	.72
Hermann	1933	1995	Louisiana	1933	1995	.73
Hermann	1933	1995	Meredosia	1933	1995	.63
Hermann	1933	1995	Alton	1933	1995	.68
Hermann	1933	1995	St Louis	1933	1995	.92
Hermann	1933	1995	Chester	1933	1995	.91

station	year	year	station	year	year	correlation
Anoka	1933	1995	St Paul	1933	1995	.88
Anoka	1933	1995	Mankato	1933	1995	.65
Anoka	1933	1995	Winona	1933	1995	.82
Anoka	1933	1995	Dubuque	1933	1995	.71
Anoka	1933	1995	Clinton	1933	1995	.69
Anoka	1933	1995	Keokuk	1933	1995	.53
Anoka	1933	1995	Hannibal	1933	1995	.50
Anoka	1933	1995	Louisiana	1933	1995	.44
Anoka	1933	1995	Meredosia	1933	1995	.29
Anoka	1933	1995	Alton	1933	1995	.50
Anoka	1933	1995	St Louis	1933	1995	.36
Anoka	1933	1995	Chester	1933	1995	.37

station	year	year	station	year	year	correlation
St Paul	1933	1995	Mankato	1933	1995	.86
St Paul	1933	1995	Winona	1933	1995	.82
St Paul	1933	1995	Dubuque	1933	1995	.71
St Paul	1933	1995	Clinton	1933	1995	.72
St Paul	1933	1995	Keokuk	1933	1995	.63
St Paul	1933	1995	Hannibal	1933	1995	.63
St Paul	1933	1995	Louisiana	1933	1995	.56
St Paul	1933	1995	Meredosia	1933	1995	.39
St Paul	1933	1995	Alton	1933	1995	.62
St Paul	1933	1995	St Louis	1933	1995	.52
St Paul	1933	1995	Chester	1933	1995	.52

station	year	year	station	year	year	correlation

Mankato	1933	1995	Winona	1933	1995	.66
Mankato	1933	1995	Dubuque	1933	1995	.57
Mankato	1933	1995	Clinton	1933	1995	.60
Mankato	1933	1995	Keokuk	1933	1995	.59
Mankato	1933	1995	Hannibal	1933	1995	.57
Mankato	1933	1995	Louisiana	1933	1995	.54
Mankato	1933	1995	Meredosia	1933	1995	.38
Mankato	1933	1995	Alton	1933	1995	.58
Mankato	1933	1995	St Louis	1933	1995	.52
Mankato	1933	1995	Chester	1933	1995	.53

Table D.2: Cross correlations between annual maximum daily flow values (continued)

station	year	year	station	year	year	correlation
Winona	1933	1995	Dubuque	1933	1995	.84
Winona	1933	1995	Clinton	1933	1995	.81
Winona	1933	1995	Keokuk	1933	1995	.58
Winona	1933	1995	Hannibal	1933	1995	.52
Winona	1933	1995	Louisiana	1933	1995	.49
Winona	1933	1995	Meredosia	1933	1995	.27
Winona	1933	1995	Alton	1933	1995	.52
Winona	1933	1995	St Louis	1933	1995	.40
Winona	1933	1995	Chester	1933	1995	.39

station	year	year	station	year	year	correlation
Dubuque	1933	1995	Clinton	1933	1995	.96
Dubuque	1933	1995	Keokuk	1933	1995	.78
Dubuque	1933	1995	Hannibal	1933	1995	.68
Dubuque	1933	1995	Louisiana	1933	1995	.55
Dubuque	1933	1995	Meredosia	1933	1995	.32
Dubuque	1933	1995	Alton	1933	1995	.52
Dubuque	1933	1995	St Louis	1933	1995	.53
Dubuque	1933	1995	Chester	1933	1995	.51

station	year	year	station	year	year	correlation
Clinton	1933	1995	Keokuk	1933	1995	.82
Clinton	1933	1995	Hannibal	1933	1995	.73
Clinton	1933	1995	Louisiana	1933	1995	.61
Clinton	1933	1995	Meredosia	1933	1995	.34
Clinton	1933	1995	Alton	1933	1995	.57
Clinton	1933	1995	St Louis	1933	1995	.58
Clinton	1933	1995	Chester	1933	1995	.56

station	year	year	station	year	year	correlation
Keokuk	1933	1995	Hannibal	1933	1995	.95
Keokuk	1933	1995	Louisiana	1933	1995	.83
Keokuk	1933	1995	Meredosia	1933	1995	.54
Keokuk	1933	1995	Alton	1933	1995	.80
Keokuk	1933	1995	St Louis	1933	1995	.76
Keokuk	1933	1995	Chester	1933	1995	.73

station	year	year	station	year	year	correlation
Hannibal	1933	1995	Louisiana	1933	1995	.89
Hannibal	1933	1995	Meredosia	1933	1995	.67

Hannibal	1933	1995	Alton	1933	1995	.88
Hannibal	1933	1995	St Louis	1933	1995	.84
Hannibal	1933	1995	Chester	1933	1995	.82

station	year	year	station	year	year	correlation
Louisiana	1933	1995	Meredosia	1933	1995	.62
Louisiana	1933	1995	Alton	1933	1995	.79
Louisiana	1933	1995	St Louis	1933	1995	.75
Louisiana	1933	1995	Chester	1933	1995	.80

Table D.2: Cross correlations between annual maximum daily flow values (continued)

station	year	year	station	year	year	correlation
Meredosia	1933	1995	Alton	1933	1995	.79
Meredosia	1933	1995	St Louis	1933	1995	.74
Meredosia	1933	1995	Chester	1933	1995	.78

station	year	year	station	year	year	correlation
Alton	1933	1995	St Louis	1933	1995	.84
Alton	1933	1995	Chester	1933	1995	.83

station	year	year	station	year	year	correlation
St Louis	1933	1995	Chester	1933	1995	.99

Table D.3: Error covariance matrix for log-Pearson III Distribution skew coefficient

station 1

1vs 1	1vs 2	1vs 3	1vs 4	1vs 5	1vs 6	1vs 7	1vs 8	1vs 9	1vs10
.0927	.0853	.0745	.0515	.0177	.0060	.0048	.0165	.0140	.0066

1vs11	1vs12	1vs13	1vs14	1vs15	1vs16	1vs17	1vs18	1vs19	1vs20
.0041	.0100	.0142	.0064	.0053	.0060	.0008	.0043	.0066	.0066

station 2

2vs 2	2vs 3	2vs 4	2vs 5	2vs 6	2vs 7	2vs 8	2vs 9	2vs10	2vs11
.0927	.0841	.0598	.0171	.0054	.0042	.0161	.0169	.0093	.0057

2vs12	2vs13	2vs14	2vs15	2vs16	2vs17	2vs18	2vs19	2vs20
.0112	.0152	.0081	.0067	.0052	-.0001	.0047	.0065	.0060

station 3

3vs 3	3vs 4	3vs 5	3vs 6	3vs 7	3vs 8	3vs 9	3vs10	3vs11	3vs12
.0946	.0697	.0231	.0123	.0100	.0157	.0204	.0133	.0050	.0128

3vs13	3vs14	3vs15	3vs16	3vs17	3vs18	3vs19	3vs20
.0159	.0133	.0113	.0080	.0016	.0068	.0118	.0111

station 4

4vs 4	4vs 5	4vs 6	4vs 7	4vs 8	4vs 9	4vs10	4vs11	4vs12	4vs13
.0901	.0476	.0325	.0225	.0146	.0262	.0197	.0066	.0135	.0167

4vs14	4vs15	4vs16	4vs17	4vs18	4vs19	4vs20
.0178	.0179	.0118	.0018	.0102	.0204	.0129

station 5

5vs 5	5vs 6	5vs 7	5vs 8	5vs 9	5vs10	5vs11	5vs12	5vs13	5vs14
.0965	.0678	.0530	.0070	.0180	.0179	.0029	.0118	.0166	.0180

5vs15	5vs16	5vs17	5vs18	5vs19	5vs20
.0189	.0189	.0082	.0126	.0375	.0258

station 6

6vs 6	6vs 7	6vs 8	6vs 9	6vs10	6vs11	6vs12	6vs13	6vs14	6vs15
.0979	.0791	.0022	.0104	.0145	.0000	.0074	.0127	.0228	.0311

6vs16	6vs17	6vs18	6vs19	6vs20
.0300	.0178	.0246	.0626	.0431

station 7

7vs 7	7vs 8	7vs 9	7vs10	7vs11	7vs12	7vs13	7vs14	7vs15	7vs16
.0982	.0021	.0085	.0107	-.0035	.0059	.0118	.0229	.0336	.0369

7vs17	7vs18	7vs19	7vs20
.0233	.0263	.0764	.0560

station 8

8vs 8	8vs 9	8vs10	8vs11	8vs12	8vs13	8vs14	8vs15	8vs16	8vs17
.0936	.0657	.0253	.0535	.0325	.0316	.0201	.0135	.0110	-.0038

8vs18	8vs19	8vs20
.0113	.0026	.0037

station 9

9vs 9	9vs10	9vs11	9vs12	9vs13	9vs14	9vs15	9vs16	9vs17	9vs18
.0986	.0607	.0500	.0296	.0314	.0255	.0230	.0128	.0004	.0218

9vs19	9vs20
.0130	.0088

station 10

10vs10	10vs11	10vs12	10vs13	10vs14	10vs15	10vs16	10vs17	10vs18	10vs19
.0907	.0195	.0104	.0129	.0181	.0157	.0105	-.0012	.0161	.0139

10vs20

.0099

station 11

11vs11	11vs12	11vs13	11vs14	11vs15	11vs16	11vs17	11vs18	11vs19	11vs20
.0941	.0519	.0468	.0173	.0092	.0072	-.0037	.0078	-.0010	-.0012

station 12

12vs12	12vs13	12vs14	12vs15	12vs16	12vs17	12vs18	12vs19	12vs20
.0896	.0810	.0429	.0273	.0143	.0070	.0100	.0111	.0082

station 13

13vs13	13vs14	13vs15	13vs16	13vs17	13vs18	13vs19	13vs20
.0937	.0523	.0355	.0217	.0084	.0151	.0186	.0142

station 14

14vs14	14vs15	14vs16	14vs17	14vs18	14vs19	14vs20
.0982	.0800	.0558	.0181	.0444	.0399	.0280

station 15

15vs15	15vs16	15vs17	15vs18	15vs19	15vs20
.0909	.0641	.0293	.0594	.0542	.0379

Table D.3: Error covariance matrix for log-Pearson III Distribution skew coefficient (continued)

station 16

16vs16	16vs17	16vs18	16vs19	16vs20
.0958	.0240	.0447	.0395	.0373

station 17

17vs17	17vs18	17vs19	17vs20
.0916	.0422	.0338	.0303

station 18

18vs18	18vs19	18vs20
.0908	.0503	.0364

station 19

19vs19	19vs20
.0954	.0686

station 20

20vs20
.0902

Table D.4: Error covariance matrix for generalized normal distribution shape factor

station 1

1vs 1	1vs 2	1vs 3	1vs 4	1vs 5	1vs 6	1vs 7	1vs 8	1vs 9	1vs10
.0064	.0060	.0050	.0034	.0012	.0006	.0004	.0010	.0009	.0004
1vs11	1vs12	1vs13	1vs14	1vs15	1vs16	1vs17	1vs18	1vs19	1vs20
.0005	.0008	.0010	.0004	.0003	.0002	.0000	.0001	.0004	.0003

station 2

2vs 2	2vs 3	2vs 4	2vs 5	2vs 6	2vs 7	2vs 8	2vs 9	2vs10	2vs11
.0064	.0057	.0041	.0012	.0007	.0004	.0010	.0011	.0006	.0005
2vs12	2vs13	2vs14	2vs15	2vs16	2vs17	2vs18	2vs19	2vs20	
.0009	.0011	.0005	.0004	.0002	-.0001	.0002	.0004	.0004	

station 3

3vs 3	3vs 4	3vs 5	3vs 6	3vs 7	3vs 8	3vs 9	3vs10	3vs11	3vs12
.0063	.0049	.0016	.0012	.0008	.0010	.0014	.0010	.0005	.0010
3vs13	3vs14	3vs15	3vs16	3vs17	3vs18	3vs19	3vs20		
.0011	.0009	.0007	.0004	.0001	.0003	.0009	.0008		

station 4

4vs 4	4vs 5	4vs 6	4vs 7	4vs 8	4vs 9	4vs10	4vs11	4vs12	4vs13
.0062	.0030	.0025	.0016	.0012	.0019	.0017	.0008	.0013	.0015
4vs14	4vs15	4vs16	4vs17	4vs18	4vs19	4vs20			
.0015	.0014	.0008	.0003	.0008	.0016	.0010			

station 5

5vs 5	5vs 6	5vs 7	5vs 8	5vs 9	5vs10	5vs11	5vs12	5vs13	5vs14
.0066	.0046	.0034	.0009	.0014	.0015	.0006	.0013	.0017	.0017
5vs15	5vs16	5vs17	5vs18	5vs19	5vs20				
.0015	.0014	.0007	.0009	.0025	.0017				

station 6

6vs 6	6vs 7	6vs 8	6vs 9	6vs10	6vs11	6vs12	6vs13	6vs14	6vs15
.0067	.0052	.0004	.0008	.0011	.0003	.0009	.0013	.0020	.0023
6vs16	6vs17	6vs18	6vs19	6vs20					
.0023	.0013	.0017	.0043	.0031					

station 7

7vs 7	7vs 8	7vs 9	7vs10	7vs11	7vs12	7vs13	7vs14	7vs15	7vs16
.0065	.0002	.0005	.0007	.0001	.0008	.0011	.0017	.0022	.0024
7vs17	7vs18	7vs19	7vs20						
.0016	.0017	.0050	.0038						

station 8

8vs 8	8vs 9	8vs10	8vs11	8vs12	8vs13	8vs14	8vs15	8vs16	8vs17
.0065	.0045	.0020	.0038	.0025	.0023	.0010	.0005	.0006	-.0002
8vs18	8vs19	8vs20							
.0005	.0001	-.0003							

station 9

9vs 9	9vs10	9vs11	9vs12	9vs13	9vs14	9vs15	9vs16	9vs17	9vs18
.0067	.0044	.0040	.0026	.0026	.0015	.0012	.0008	.0001	.0013
9vs19	9vs20								
.0008	.0001								

station 10

10vs10	10vs11	10vs12	10vs13	10vs14	10vs15	10vs16	10vs17	10vs18	10vs19
.0063	.0018	.0011	.0012	.0011	.0009	.0007	.0002	.0012	.0010
10vs20									
.0006									

station 11

11vs11	11vs12	11vs13	11vs14	11vs15	11vs16	11vs17	11vs18	11vs19	11vs20
.0066	.0042	.0037	.0014	.0008	.0009	.0002	.0007	.0001	-.0006

station 12

12vs12	12vs13	12vs14	12vs15	12vs16	12vs17	12vs18	12vs19	12vs20
.0065	.0057	.0029	.0018	.0013	.0006	.0007	.0008	.0002

station 13

13vs13	13vs14	13vs15	13vs16	13vs17	13vs18	13vs19	13vs20
.0067	.0034	.0023	.0015	.0007	.0010	.0012	.0005

station 14

14vs14	14vs15	14vs16	14vs17	14vs18	14vs19	14vs20
.0067	.0056	.0038	.0013	.0033	.0029	.0019

station 15

15vs15	15vs16	15vs17	15vs18	15vs19	15vs20
.0062	.0044	.0018	.0040	.0035	.0024

Table D.4: Error covariance matrix for generalized normal distribution shape factor (continued)

station 16

16vs16	16vs17	16vs18	16vs19	16vs20
.0065	.0015	.0029	.0025	.0024

station 17

17vs17	17vs18	17vs19	17vs20
.0064	.0028	.0021	.0021

station 18

18vs18	18vs19	18vs20
.0064	.0034	.0026

station 19

19vs19	19vs20
.0064	.0048

station 20

20vs20
.0074

Table D.5: Error covariance matrix for generalized extreme value distribution shape factor

station 1

1vs 1	1vs 2	1vs 3	1vs 4	1vs 5	1vs 6	1vs 7	1vs 8	1vs 9	1vs10
.0087	.0082	.0068	.0045	.0014	.0007	.0003	.0010	.0010	.0004

1vs11	1vs12	1vs13	1vs14	1vs15	1vs16	1vs17	1vs18	1vs19	1vs20
.0004	.0008	.0010	.0003	.0001	.0002	.0002	.0002	.0003	.0004

station 2

2vs 2	2vs 3	2vs 4	2vs 5	2vs 6	2vs 7	2vs 8	2vs 9	2vs10	2vs11
.0090	.0079	.0056	.0015	.0006	.0003	.0011	.0013	.0007	.0006

2vs12	2vs13	2vs14	2vs15	2vs16	2vs17	2vs18	2vs19	2vs20
.0009	.0011	.0004	.0003	.0002	.0001	.0002	.0004	.0004

station 3

3vs 3	3vs 4	3vs 5	3vs 6	3vs 7	3vs 8	3vs 9	3vs10	3vs11	3vs12
.0087	.0067	.0019	.0011	.0008	.0010	.0015	.0009	.0005	.0010

3vs13	3vs14	3vs15	3vs16	3vs17	3vs18	3vs19	3vs20
.0011	.0008	.0006	.0002	.0001	.0003	.0008	.0008

station 4

4vs 4	4vs 5	4vs 6	4vs 7	4vs 8	4vs 9	4vs10	4vs11	4vs12	4vs13
.0085	.0039	.0028	.0019	.0011	.0021	.0017	.0008	.0012	.0014

4vs14	4vs15	4vs16	4vs17	4vs18	4vs19	4vs20
.0015	.0013	.0007	.0003	.0009	.0018	.0010

station 5

5vs 5	5vs 6	5vs 7	5vs 8	5vs 9	5vs10	5vs11	5vs12	5vs13	5vs14
.0093	.0063	.0046	.0006	.0013	.0014	.0005	.0012	.0017	.0019

5vs15	5vs16	5vs17	5vs18	5vs19	5vs20
.0017	.0017	.0007	.0012	.0035	.0020

station 6

6vs 6	6vs 7	6vs 8	6vs 9	6vs10	6vs11	6vs12	6vs13	6vs14	6vs15
.0092	.0069	.0000	.0006	.0010	.0000	.0006	.0010	.0023	.0030

6vs16	6vs17	6vs18	6vs19	6vs20
.0028	.0014	.0023	.0059	.0038

station 7

7vs 7	7vs 8	7vs 9	7vs10	7vs11	7vs12	7vs13	7vs14	7vs15	7vs16
.0091	.0000	.0005	.0007	-.0001	.0006	.0010	.0020	.0029	.0032

7vs17	7vs18	7vs19	7vs20
.0020	.0023	.0069	.0047

station 8

8vs 8	8vs 9	8vs10	8vs11	8vs12	8vs13	8vs14	8vs15	8vs16	8vs17
.0090	.0055	.0019	.0049	.0031	.0028	.0010	.0003	.0006	-.0003

8vs18	8vs19	8vs20
.0004	-.0001	-.0002

station 9

9vs 9	9vs10	9vs11	9vs12	9vs13	9vs14	9vs15	9vs16	9vs17	9vs18
.0087	.0053	.0050	.0030	.0031	.0017	.0013	.0008	.0001	.0015

9vs19	9vs20
.0008	.0003

station 10

10vs10	10vs11	10vs12	10vs13	10vs14	10vs15	10vs16	10vs17	10vs18	10vs19
.0082	.0017	.0009	.0011	.0013	.0010	.0006	.0003	.0012	.0011

10vs20
.0006

station 11

11vs11	11vs12	11vs13	11vs14	11vs15	11vs16	11vs17	11vs18	11vs19	11vs20
.0091	.0052	.0045	.0013	.0006	.0008	.0000	.0004	-.0001	-.0005

station 12

12vs12	12vs13	12vs14	12vs15	12vs16	12vs17	12vs18	12vs19	12vs20
.0087	.0075	.0034	.0019	.0011	.0007	.0006	.0008	.0002

station 13

13vs13	13vs14	13vs15	13vs16	13vs17	13vs18	13vs19	13vs20
.0087	.0040	.0025	.0016	.0007	.0009	.0012	.0005

station 14

14vs14	14vs15	14vs16	14vs17	14vs18	14vs19	14vs20
.0087	.0073	.0048	.0013	.0043	.0037	.0022

station 15

15vs15	15vs16	15vs17	15vs18	15vs19	15vs20
.0087	.0059	.0020	.0054	.0049	.0031

Table D.5: Error covariance matrix for generalized extreme value distribution shape factor (continued)

station 16

16vs16	16vs17	16vs18	16vs19	16vs20
.0088	.0017	.0038	.0034	.0030

station 17

17vs17	17vs18	17vs19	17vs20
.0083	.0032	.0026	.0022

station 18

18vs18	18vs19	18vs20
.0090	.0050	.0034

station 19

19vs19	19vs20
.0093	.0061

station 20

20vs20
.0090

Appendix E: Interagency and Technical Advisory Group Recommendations

E.1 Introduction

The selection of a distribution/estimation pairing was based on recommendations and review comments from the Interagency and Technical Advisory Groups (IAG and TAG, see section 1 for membership). Although some consensus was reached on the selection, members were not in total agreement on all aspects of the final recommendations. This is probably not too surprising given the diverse backgrounds and experience of the group members.

Section E.2 provides a copy of a memorandum describing recommendations regarding the selection of a distribution/estimation procedure; as well as recommendations on some other ancillary topics. This memorandum reflects a not easily reached consensus. Section E.3 provides some quotes from some of the members which provides a picture of the diversity of opinion within the group and the fact that not all members were entirely comfortable with the final recommendations. The members did feel the selection of the log-Pearson III distribution was pragmatic, but not necessarily the best approach. There was more disagreement expressed with regard to the application of regional skew values.

In reviewing the recommendations, recognize the selection of the log-Pearson III distribution/standard method of moments was made because difference with other methods was not practically significant. Other methods tested certainly are adequate for computing flood quantiles. However, the IAG and TAG did not wish to recommend deviation from the standard practice given the computed differences between methods.

E.2 Memorandum summarizing IAG and TAG recommendations

Memorandum for record 6/1/99

to: Technical Advisory Group (TAG), Interagency Advisory Group, (IAG), Upper Mississippi Basin Flood Frequency Study (TAG members: Jon Hosking (IBM), William Lane (Consultant, Denver, CO), Kenneth Potter (University of Wisconsin), Jery Stedinger (Cornell University), Wilbert Thomas (Michael Baker Corporation), IAG members: Kenneth Bullard (Bureau of Reclamation), Alan Johnson (Federal Emergency Management Agency), Lesley Julian (National Weather Service), William Kirby (Geological Survey) and Don Woodward (Natural Resource Conservation Service)

from: David Goldman

subject: Meeting summary, April 8th, 1999, held at Michael Baker Corporation, Alexandria, VA.

Executive Summary

The technical advisory group conclusions with regard to obtaining a flood distribution/estimation procedure are as follows:

1. The committee makes these recommendations under the assumption that the Corps will have obtained a significantly long period unregulated flow record of 110 years for estimating flood quantiles. The recommendations to use the following approach depends on this assumption.

2. The differences in flood quantiles obtained with different distribution/estimation pairings does not warrant deviation from the basic Bulletin 17B methodology, i.e., the log-Pearson III distribution estimated using the method of moments applied to flow logarithms.

3. The expected moments algorithm (Cohn, et al., 1997) is superior to and should be used in place of the existing Bulletin 17B method for distribution estimation in the presence of low outliers or historical information.

4. An adopted skew value can be computed using the standard bulletin 17B weighting of station and regional skew values; where the regional skewness coefficient and mean square error for the regional skew is obtained from a generalized least squares analysis.

5. Regionally consistent flood quantiles will be obtained for gage sites by averaging, if necessary, skew values for river reaches between major confluences.

6. Flood distributions statistics between sites will be obtained by linearly interpolating the mean, standard deviation and skew coefficient for log-flows with drainage area.

7. Historical estimates of floods are likely to be significantly less accurate then the systematically observed floods, and the systematic record is over a hundred years in length. The consistency of flood distributions estimated from the systematic period of record with historical information needs to be evaluated. If the estimates are inconsistent, then some judgement will need to be made with regard to the potential of the historical data, given its limitations, for improving flood distribution estimates obtained from a systematic period of significant length.

The interagency advisory group concurred with these findings for the most part. This group had the following comments and concerns:

8. The key problem in estimating distributions given historical information and low outliers is in judging the worth of the historical information and the censoring level for the low outliers.

Other issues addressed by the technical advisory group:

9. Analysis of some maximum annual gage records reveal a statistically significant trend with time. The Corps needs to determine if these trends are an artifact of regulation, flow measurement corrections or some more fundamental cause (land use change, influence of climate). If the trends are real then additional thought needs to be given to the estimation of flood distributions.

10. The period of record simulations might be performed with no levee failures due to overtopping. Levee performance evaluated in this manner would remove the influence of failure timing on the estimate of regulated flood distributions. In addition, the value of this approach is in avoiding the need to redo the analysis whenever a levee is upgraded.

11. Conversion of daily to peak annual flood distributions is not likely to be a significant problem for the study. However, when necessary, the conversion should be based on a regression between ranked observed daily and peak flow values.

1. Introduction

The primary purpose of the meeting was to finalize the Technical Advisory Group (TAG) and Interagency Advisory Group (IAG) recommendations with regard to selection of the flood distribution/estimation methodology to be used in the subject study. Other issues discussed were with regard to trend analysis, modeling levees, and converting daily to peak discharges.

Section 2 describes the discussion and recommendation for estimating the flood distribution for the record of systematically observed unregulated flow values. The methods recommended for smoothing distribution statistics and obtaining consistent flood quantiles along study area reaches is discussed in section 3. Section 4 describes the issue with regard to incorporating historical information into distribution estimates. The interagency groups concerns with regard to the TAG recommended distribution/estimation methods is summarized in section 5.

Although arriving at the recommended distribution/estimation procedure was the primary goal of the meeting, the other issue discussed related to the derivation of regulated and stage frequency curves. Trends or other indicators of non-randomness were investigated to determine if land use change or some type of climatic variability might violate the assumption of randomness in the unregulated flood series needed for inferring a flood distribution. Section 6 describes concerns about the importance of trends resulting from some of the work performed at the Institute for Water Resources (Olsen and Stakhiv, 1999).

Flood distributions estimates will be used to derive the stage frequency curves for the study area reaches. The derivation will be performed by initially converting the unregulated flood distributions to regulated distributions using a relationship between unregulated and regulated flows. Then the regulated distributions are converted to a stage distribution using a rating curve. An unsteady flow model is used to both establish the relationship between unregulated and regulated flows, and the rating curve. Section 7 discusses the issues raised with regard to a key aspect of the unsteady flow modeling, the assumptions regarding levee performance. Finally, section 8 describes the discussion on estimating peak flow distributions from daily estimates. This is probably a minor issue for most of the study area, but may be of concern for the most upstream gages.

The following summarizes the recommendations of the TAG members, except Jon Hosking, and, IAG members, except Don Woodward, who could not be present due to prior commitments. There comments with regard to the overall recommendations will be enlisted in the near future.

2. Selection of distribution/estimation method

2.1 Background

The IAG and TAG have disagreed on the worth of a regional shape factor or skew for the study area. The IAG recommended against the use of a regional skew coefficient (minutes of second Task Force Meeting, Upper Mississippi Flood Frequency Study, St. Louis) in a distribution estimation procedure because of: 1) the great variation in meteorology; 2) diverse nature of the hydrologic responses and; 3) the high correlation of flooding between gages in the study area. Given this recommendation, the Hydrologic Engineering Center (HEC, 1998) compared the at-site estimates of flood quantiles obtained from different distribution/estimation pairings. The comparison revealed an average difference between the log-Pearson III distribution estimated by method of moment application to log-flows and the other method tested of about 10%.

The TAG reviewed the comparison results (second meeting of the Technical Advisory Group (minutes of April 15th, 1998, meeting held at Michael Baker Corporation, Alexandria, VA, edit by Hydrologic Engineering Center, Davis, CA) and still felt that regional information had something to offer in terms of improving estimates of flood distributions. They recommended different distribution/ estimation techniques employing both regional shape and at-site estimation be compared to the standard Bulletin 17B methodology to see if there is a significant difference. These analyses were made and reported in HEC(1999) which was review by the TAG and IAG.

This document describes the first joint meeting of the IAG and TAG held to discuss these disagreements and come to some mutual conclusions with regard to recommendations for the flood distribution estimation techniques for the study.

2.2 Results of comparative testing recommended by the TAG

The following distribution/estimation pairings were investigated based on the TAG recommendations (see HEC (1999)):

$ Bulletin 17B (IACWD, 1982) (log-Pearson III distribution, method of moments estimations; adopted skew obtained from weighting regional and at-site skew values);
$ Bulletin 17B, except adopted skew equated to regional skew (regional shape estimation);
$ Bulletin 17B, except adopted skew equated to regional skew, regional shape obtained from generalized least squares regression (regional shape estimation);
$ application of log-Pearson III distribution to censored data sets, censor data less than median , distribution estimates using Expected Moments Algorithm (EMA) (at-site estimation);
$ application of log-Pearson III distribution to censored data sets, censor data less than median , distribution estimates using Expected Moments Algorithm (EMA), adopted skew equated to regional skew (regional shape estimation) ;
$ application of log-Pearson III distribution to censored data sets, censor data less than median , distribution estimates using Expected Moments Algorithm (EMA), adopted skew equated to regional skew, regional skew obtained from generalize least squares regression (regional shape estimation);
$ generalized normal distribution, parameters estimated using L-moments (at-site estimation);
$ generalized extreme value distribution, parameters estimated using L-moments (at-site

estimation);

- $ generalized normal distribution, parameters estimated using L-moments, shape parameter equated to regional shape value (regional shape estimation);
- $ generalized extreme value distribution, parameters estimated using L-moments, shape parameter equated to regional shape value (regional shape estimation);
- $ generalized normal distribution, parameters estimated using L-moments, shape parameter equated to regional shape value, regional shape obtained from generalized least squares regression(regional shape estimation);
- $ generalized extreme value, parameters estimated using L-moments, shape parameter equated to regional shape value, regional shape obtained from generalized least squares regression(regional shape estimation.

The selection of the generalized normal distribution resulted from the estimation methodology described in Hosking and Wallis (1997). The generalized extreme value distribution was applied per the suggestion of the TAG because this distribution has recently received considerable attention (for further details on the selection procedure see HEC, 1998).

The regional shape or regional skew parameters in regional shape estimation were to be obtained by either obtaining the average value for study area regions defined as being homogenous in application of the previously mentioned Hosking and Wallis estimation procedure; or, by the application of generalized least squares (GLS) regression (Tasker and Stedinger, 1986, 1989). However, the GLS applications resulted in regression equations which effectively predicted shape and/or skew values that were constant over the region (see HEC, 1999, Appendix C). Consequently, constant values of regional shape or skew estimates were only used in the comparisons.

Comparisons with the Bulletin 17B method, the method recommended for federal agency application, resulted in a maximum average difference of 10-15% in the 1% chance exceedance flow value. The maximum difference occurred in comparisons with distributions estimated using regional shape values. These differences were somewhat worrisome. However, comparison of at-site estimates of distribution did not cause great differences, except at one station (Mankato). Consequently, the TAG felt the log-Pearson III was sufficiently flexible to describe the distribution of flood flows in the basin, and was consistent with Bulletin 17B. Consequently, the log-Pearson distribution estimated using the method of moments applied to log-flows was considered to be adequate for obtaining flood distributions.

However, the TAG (Stedinger, Potter, Lane) did recommend application of the EMA for estimating the distribution when low-outliers are detected or when there is historical information available instead of the estimator recommended in Bulletin 17B. The TAG is not comfortable with the current guidelines application of the conditional probability adjustment to data sets censored for low outliers. The TAG (Stedinger) observed that with distributions like those for the lower reaches of the upper Mississippi with very negative log-space skewness coefficients, there is perhaps a 50% probability that a low outlier will be detected in a sample by the Bulletin 17B low-outlier test (IACWD, 1982, Appendix 4). Low outliers were in fact detected at most gauges in the lower part of the basin. This is not a problem if such samples are processed well. However, if the lowest observation(s) are dropped, and a Pearson type 3 distribution is fitted to the logarithms of the remaining data, it is not clear that the resulting skewness coefficient is compatible with a Bulletin 17B map skew, or skewness coefficients computed from complete samples at other sites. Thus the TAG accepted the recommendation that when low outliers are detected, that the censored sample be analyzed with the EMA (threshold set at the largest observation retained), and not the conditional probability model suggested in Bulletin 17B (IACWD, 1982, paragraph #9, p. 18). The recently developed EMA fits a LP3 distribution to the original data set subject to any specified censoring and thus represents a treatment of the data consistent with the basic Bulletin 17B method of moments procedure. The TAG (Potter) also observed that EMA is more consistent with the Bulletin 17B moments philosophy for parameter estimation than the historical weighted moments procedure that was included in the Bulletin.

3. Regional smoothing

Regional smoothing of the log-Pearson III statistics is necessary to obtain consistent estimates of distribution frequency curves along the study area reaches. A difficulty with performing the smoothing is the potential for abrupt changed in statistics, particularly skew coefficient at the confluence with a major tributary. This difficulty will be resolved by using the results of the unsteady flow modeling to determine if the confluence does have a significant impact on peaks of major flood values; and if so, what is the upstream and downstream boundaries of this influence. A smoothing algorithm will be applied between boundaries, if in fact these boundaries can be identified.

Smoothing algorithms were postulated to both revise at-site (i.e., at gage locations) estimates and to obtain estimates between gages. The three possible smoothing algorithms suggested for the at-site estimates were as follows:

1. Examine predicted quantiles (e.g., the 1% chance exceedance flow value) for consistency along study area reaches. A glaring inconsistency, for example, would occur when an upstream estimate of the 1% chance discharge exceeded a downstream value. Under these circumstances, quantiles would be revised based on judgment, and flood distribution statistics would be adjusted to reproduce these revised estimates.

2. Estimate an average skew coefficient for study area reaches, smooth the mean using a regression with drainage area and use either an average standard deviation or the at-site value.

3. Obtain an adopted skew coefficient as a weighted average of the at-site value and a regional average, and use the at-site mean and standard deviation. The regional and at-site skew values will be weighted inversely to the estimated mean square error as described in Bulletin 17B. The mean square error for the at-site skew value is computed based on record length and the mean square error of the regional skew is obtained from GLS estimates. The TAG recommended selection of (3) as being most consistent with the Bulletin 17B guidelines. However if inconsistent flood profile occur because of sampling variability, then an average skew value for the reach should be chosen as in (2).

Two methods suggested for obtaining flood distribution statistics between gages:

4. Period of record simulations with the unsteady flow model will provide estimates of peak annual daily flow values at each study area location of interest. Estimates of distribution statistics from these simulated flows can be used to obtain smoothed estimates between gages.
5. Linear interpolation of the mean, standard deviation and skew between smoothed at-site estimates.

The TAG preferred 5 as being easier to explain and simpler in application. The issue of what to do if only one gauge is located within a reach was not discussed which would make it difficult to interpolate or extrapolate the quantiles from the gauged site to other locations within the reach. In this case, it may be necessary for the USACE to use results of the unsteady flow model to translate quantiles to other locations within the reach.

4. Application of historical information

Historical information, i.e. observations of floods outside the period covered by the systematic period, spanning different periods of record and of variable quality is available throughout the study area. The approaches suggested for using this information are:

$ Best estimates of historical information will be used to improve at-site estimates. Some type of multivariate technique (possibly similar to the two-station comparison described in Bulletin 17B) will need to be developed to obtain improved estimates of parameters at other sites where either historical information is not available or does not span the same period of record.

$ Examine if the frequency curve obtained from the at-site statistics is consistent with the frequency curves obtained with various estimates of the historical information. The accuracy of or confidence in the historical information will determine the range in magnitudes.

Historical flood information is likely to be available at a few sites. The use of this information at St. Louis, and perhaps less information at other sites, was discussed. The upper Mississippi River and Missouri basin study is particularly challenging because of its size, the complexity of the hydraulics, and the range of climatic regions included within the drainage area of the many large tributaries of the Mississippi and Missouri Rivers. A common period of record at almost all sites was adopted so as to be able to develop a consistent set of flood frequency relationships. It was feared that use of historical information at only a very few sites would cause problems. Moreover, the systematic records are already in excess of 100 years. In addition, it was not clear to the TAG that estimates of the flow rates for historical peaks before 1860 would have sufficient precision to add much information to the flood frequency analysis. Because of the instability of the channel at that time, the shallow slope of the river, and the wide flood plain, it is very difficult to estimate flood flow rates with much precision without great effort. Thus the TAG recommended (2, see section 3). Where there is inconsistency, then (1, see section 3) might be considered.

5. Interagency group comments

The interagency group was in general agreement with the proposed method for flood distribution estimation. This group did have the following concerns over some of the recommendations:

$ The key problem in estimating distributions is in determining the censoring level for the low-outliers and evaluating the worth of the historical information. The estimation procedure, EMA or as described in Bulletin 17B is of secondary importance.

$ Smoothing of statistics will tend to reduce the apparent fit between distribution predictions and plotting positions of the observed data. There was some concern that reducing the positive skew values may tend to reduce flood quantiles more than is advisable (Bullard). The end users of the flood distribution estimates might find this disturbing not being familiar with the statistical problems caused by sampling error. A particular difficulty may occur if the distribution consistently under or over predicts the top ranked events (Johnson).

The TAG (Stedinger) generally felt that adjusting negative skew values upward might have a more important impact and could generate more realistic flood quantile estimates. An apparent over or under prediction of top-ranked events is likely because of the high correlation between peak annual stream flow values and should not be a concern (Lane).

6. Trend analysis

The TAG (Stedinger) shared a number of results from IWR (1999). A very large and statistically significant trend was found in the Hannibal gauge record. Statistically significant trends were also found just downstream in the St. Louis gauge record, and at Herman on the Missouri River above the confluence of the Mississippi and Missouri Rivers. Newer results presented at the meeting for the Illinois River ($p = 0.7\%$), St. Croix ($p = 0.3\%$), and the Minnesota River ($p = 2.8\%$) showing statistically significant positive trends with time. There does not seem to be much regulation on these rivers. The Illinois River is near Hannibal and also represents a very large drainage area. Thus significant trends were observed in the data for Hermann on the Missouri, Hannibal on the Mississippi, and at Meredosia on the Illinois River, all above St. Louis. Regulation should result in a decrease in flood peaks, but an increase was seen. Regulation may indeed have effected some of the flood peaks so that the USGS records of observed flows are too low.

Potter (1991) shows that since 1951, flood peaks have decreased in some small agricultural catchments in Wisconsin. One can perhaps see that pattern at the end of the St. Croix record, which otherwise has an increasing trend.

Trends do not appear in the gauge record for St. Louis published by the USGS. They do after the adjustments made by the USACE.

The TAG was very concerned about these issues and believed they needed to be resolved. HEC (1998, 1999) is based on the USACE data set, not the USGS records. There was no time to discuss how frequency analysis should be conducted if records exhibited real trends.

7. Modeling of levees in the unsteady flow model

An unsteady flow model of the study area rivers will be used to perform a period of record simulation to obtain both unregulated and regulated flow values. A key aspect of this simulation is the modeling of levee performance. The current approach to including levee performance is to fail the levee whenever the simulated water surface elevation exceeds the top of the levee. Levees will be assumed to be rebuilt after a failure occurs prior to the next flood.

The TAG (Potter) suggested considering model simulations where levees are not allowed fail. This avoids the problem of having computed water surface profiles, and corresponding stage frequency curves that are influenced by the timing of levee failures. Presumably, the levee system analyzed under this scenario would reflect a well established federal levee system.

8. Converting daily to peak flow frequency curves

The conversion of daily to peak flow frequency curves is not likely to be a significant problem given the large study drainage areas. However, a significant difference exists between peak and daily maximum flow values for some of the most upstream gages. A conversion between frequency curves can probably be made by using a regression between available between daily and peak flows. Application of the regression will be not result in any significant variance reduction presuming that the R^2 is large, which, based on experience is likely. However, the TAG (Lane) suggested avoiding potential problems by developing a conversion from equating ranks of observed daily and peak flows rather than the magnitudes. It was observed that this is how the National Weather service makes adjustments to go between quantiles of the daily [fixed-24-hour period] rainfall distributions, to quantiles of the maximum 24-hour rainfall depth distributions. The ratio is about 1.13.

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E.3 Selected comments from IAG and TAG members

The goal of selecting a best distribution/estimation procedure may have been somewhat Quixotic from the very beginning. As Dr. William Kirby (1999, IAG member) pointed out:

> *I get a sense that there is some hope in the various TAGs & IAGs and Corps offices that statistics will help us get better (ie, more accurate) flood-frequency estimates. I think that this is a vain hope. There is no statistical or probabilistical theory (that I know of) that contains any hydrologic substance to help us define the form of the flood-freq distribution or the optimum method of estimation of it. There is nothing but curve-fitting (and maybe a little qualitative judgmental fudging) to the available data. What statistics CAN do for us—and it's a valuable contribution—is to give us an idea of how far off from the true distribution both our DATA and our estimates can be.*

In many respects, the results of the selection reflect these comments. The comparison made between distribution/estimation pairings and empirical distribution obtained from the data was not definitive. The sensitivity analysis resulted in average differences of about 10% over all gages investigated for the 1% event. This difference is small in comparison to the sampling error in the estimated 1%flood, even given the relatively long record lengths available at the study area gages.

An opposing view point was expressed by Dr. Jon Hosking (TAG member) who believes that combined regional shape/at-site L-moment distribution estimates are superior to estimates obtained using Bulletin 17B methods. This viewpoint was expressed in his comments with regard to the recommendations presented in the previous section. His dissenting comments are repeated here in full:

Selection of Distribution/Estimation method

I dissociate myself from assertion 1. on page 1 [note: this refers to recommendations provided in section E.2].

The technical facts are:(a) the difference in quantile estimates at the 0.01 exceedance probability does not exceed 15% at any site and is typically less than 10%;

(b) using regional shape estimation and the method of L-moments, the estimated RMSE of quantile estimates at the 0.01 exceedance probability is in the range 15-25% for plausible data generating mechanisms [see sections 5.2 and 6.4];

(c) regional shape estimation is more accurate than at-site estimation, using the method of L-moments, for plausible data generating mechanisms;

(d) the estimated RMSE of quantile estimates, or any other measure of accuracy, for the Bulletin 17B procedure is not known, but there is no reason to suppose that it is more accurate than the method of L-moments;

(e) in particular, the methods of L-moments does not require logarithmic transformation of the data and does not require special treatment of low outliers.

These facts indicate to me that regional shape estimation using L-moments can be expected to give more accurate quantile estimates, on average over all the sites, than the Bulletin 17B procedure, though the increase in accuracy may not be large.

On technical grounds alone, therefore, I recommend that frequency estimation be based on regional shape estimation using L-moments. On grounds of familiarity, or in order not to "deviate from the basic Bulletin 17B methodology", the Corps may reasonably prefer to use an approach based on Bulletin 17B instead. However, this is not a technical but a political decision, and should be acknowledged as such.

Regional Smoothing

The Corps's generalized least analysis indicates that the skewness of log-flows contains no between-site variation that cannot be explained by sampling variability. This is in agreement with my earlier analysisof the L-skewness of untransformed flows (my comments of September 14,1998, Exhibit 8) [see reference: Hosking, J.R.M., 1998. Upper Mississippi River Flood Frequency Review, Comments]. Thus I see no reason to use an adopted skew that involves weighting at-site and regional skew estimates: rather, a single regional skew estimate should be used throughout. A reasonable alternative is that the skew coefficient should be taken as constant within each reach of a river, with adjustment being made for the effect of major confluences where this can be quantified by the use of an unsteady flow model. Therefore, of the alternatives on page 8, I recommend 2. rather than 3. In the conclusions on page 1, rather than item 3. I recommend the adoption of a locally constant skew, as described above; and for item 5., interpolation of mean and standard deviation with drainage area is reasonable, while the use of a locally constant skewness would obviate the need to interpolate it between sites.

Fit between distribution predictions and plotting positions (page 2, item 8.) [note: this refers to recommendations provided in section E.2]

The Corps has done well to appreciate the utility of statistical data analysis and the effect of sampling error on the accuracy of distributions fitted to streamflow data. The IAG seems not to have this appreciation, or is concerned that end users may not. If the Corps does not want to be restricted to merely plotting data on probability paper and drawing a line through the points, some education of the IAG or the end users may be required. Perhaps some formal course could be developed, or an educational web site set up, that would enable users to familiarize themselves with the idiosyncrasies of sampling variability.

Note that bracketed comments are provided for clarifying the quoted references.

A perpsective on the contrasting viewpoints represented by Dr. Kirby and Dr. Hosking can be gained by considering the comments by Dr. Kenneth Potter (1999, TAG member) with regard to the recommendations provided in section E.2:

GENERAL COMMENTS

It is clear from the TAG comments that among the members there are conflicting philosophies regarding flood analysis. Jon Hosking and Jery Stedinger tend to support practices supported by theory and Monte Carlo simulations. Bill Kirby and Will Thomas appear to have less faith in theoretical and Monte Carlo results because of skepticism about the true distribution of floods. They rely more on empirical results and are concerned about breaking with traditional methods in the absence of compelling reasons. There are sound reasons for embracing either perspective.

SPECIFIC COMMENTS

Hosking’s Concern about Choice of the LP-3

I agree with Jon Hosking's "technical facts." The reason for choosing the LP-3 has mostly to do maintaining the traditional approach. I don't disagree with the decision, but I agree with Jon that we should state that the reason for doing so are pragmatic.

Hosking's Concern about Regional Smoothing

I am sympathetic to Jon Hosking's concern about using a weighted skew given the lack of evidence for variations in skew in space. However, I have concerns about his suggestions to use the hydraulic simulations to handle junctions as it will be very difficult to account for variations in tributary timing. My recommendation is to consider both methods 2 and 3, and choose the one that appears most reasonable.

Misinterpretation of these statements is possible. However, a reasonable assessment of the dissenting opinions is that the TAG accepts the decision to use the log-Pearson III distribution as being adequate, although this would not be the preferred choice by all the members. For example, Hosking does not expect a great increase in accuracy from the L-moment/regional shape approach over log-Pearson III/standard moments.

The method for using regional skew is more contentious. The recommendation here is for the Corps to examine a number of different methods mentioned for estimating skew. The various methods mentioned may not result in great difference between flood quantiles of interest. In any case, the skew estimation method selected not only needs to follow a statistically reasonable procedure; but also result in consistently varying flood quantiles along study area rivers.